Finite Sample Multivariate Structural Change Tests with Application to Energy Demand Models[1]

Jean-Thomas Bernard [2]
Université Laval

Nadhem Idoudi[3]
Hydro-Québec

Lynda Khalaf[4]
Université Laval

Clément Yélou[5]
Université Laval

September 21, 2004

[1] This work was supported by the Chair on the Economics of Electric Energy, the Social Sciences and Humanities Research Council of Canada, the Fonds FQRSC (Government of Québec), the Canadian Network of Centres of Excellence [program on *Mathematics of Information Technology and Complex Systems* (MITACS)], and the Institut de Finance Mathématique de Montréal. The authors thank participants at the 44th Annual Congress of the Société Canadienne de Science Économique (Québec) and the MITACS 5th Annual Conference at Dalhousie University (Halifax), for several comments and suggestions.

[2] Holder of the Chair on the Economics of Electric Energy; *Groupe de recherche en économie de l'énergie, de l'environnement et des ressources naturelles* [GREEN], Université Laval. Mailing address: Pavillon J.-A.-De Sève, St. Foy, Québec, Canada, G1K 7P4. TEL: (418) 656 5123; FAX: (418) 656 7412; Email: jtber@ecn.ulaval.ca.

[3] Conseiller Coûts et caractéristiques de la consommation, Direction des Affaires réglementaires et tarifaires, *Hydro-Québec* Distribution, and *Groupe de recherche en économie de l'énergie, de l'environnement et des ressources naturelles* [GREEN], Université Laval. Mailing address: 75, boul. René-Lévesque ouest, 2e étage, Montréal, (Québec) H2Z 1A4. Tel: (514) 289-2211 poste 6156. FAX: (514) 289-5624. Email: idoudi.nadhem@hydro.qc.ca.

[4] Corresponding author. *Groupe de recherche en économie de l'énergie, de l'environnement et des ressources naturelles* [GREEN], Université Laval. Mailing address: Pavillon J.-A.-De Sève, St. Foy, Québec, Canada, G1K 7P4. TEL: (418) 656 2131-2409; FAX: (418) 656 7412; Email: lkha@ecn.ulaval.ca.

[5] *Groupe de recherche en économie de l'énergie, de l'environnement et des ressources naturelles* [GREEN], Université Laval. Mailing address: Pavillon J.-A.-De Sève, St. Foy, Québec, Canada, G1K 7P4. Email: cyelou@ecn.ulaval.ca.

Abstract

This paper considers finite sample motivated structural change tests in the Multivariate Linear Regression model with application to energy demand models, in which case commonly used structural change tests remain asymptotic. As in Dufour and Kiviet (1996), we account for intervening nuisance parameters through a two-stage maximized Monte Carlo test procedure. Our contributions can be classified in five categories: (i) we extend tests for which a finite-sample theory has been supplied for Gaussian distributions to the non-Gaussian context; (ii) we show that Bai, Lumsdaine and Stock (1998)'s test severely over-rejects and propose two exact variants of this test; (iii) we consider predictive break test approaches which generalize the test of Dufour (1980) and Dufour and Kiviet (1996); (iv) we use a dummy-variable method to define exact (non-Bonferonni based) extensions of the multivariate outliers test proposed by Wilks (1963) to models with covariates; (v) we apply these test procedures to the energy demand system analyzed by Arsenault, Bernard, Carr and Genest-Laplante (1995). The procedures we propose have potential useful applications in statistics, econometrics and finance (*e.g.* event studies).

Key words: structural stability; structural change; multivariate linear regression model; Monte Carlo test; exact test.

Journal of Economic Literature classification: C12, C15, C30, Q4.

Contents

List of Tables

1 Introduction

Structural stability is one of the most enduring econometric problems.[1] This paper considers finite sample motivated structural change tests in the Multivariate Linear Regression (MLR) model, with application to energy demand models. The MLR model is certainly one of the most fundamental models in econometrics, statistics and finance [see Rao (1973, Chapter 8), Anderson (1984, chapters 8 and 13), Kariya (1985), Dufour and Khalaf (2002*b*), Stewart (1997) and the references therein]. In particular, energy demand analysis is often conducted within this framework, which is typically considered to estimate share equations [see e.g. Berndt (1991, Chapters 7 and 9) and Stewart (1997))].[2]

The existing literature on multivariate structural stability tests (see e.g. Bai et al. (1998), Qiu and Hawkins (2001)) is sparse compared to the univariate case, although several available procedures including e.g. Cantrell, Burrows and Vuong (1991) and Andrews (1993)'s tests are derived in a sufficiently general framework which includes the MLR as a special case. Relevant results may be found in statistical work on multivariate outlier detection, although the latter typically apply to the location-scale model without co-variates; see e.g. Hadi (1992), Qiu and Hawkins (2003), Caroni and Prescott (1992), Thode (2002, Chapter 10). The problem is also related to event studies [Binder (1985*a*), Binder (1985*b*), Schipper and Thompson (1985)]. Event studies describe financial returns via MLR equations which include, in addition to characteristic explanatory factors, dummy variables introduced to account for the event under study; the problem consists in assessing the significance of these dummies. Unfortunately the econometric literature on multivariate structural change, the financial literature on event studies and the statistics literature on multivariate outliers have remained somewhat disconnected, although the latter problems are practically and fundamentally related. In this paper, we adopt a unified testing approach relevant to these research fields.

Exact multivariate break test procedures are available for a few special cases. Hooper and Zellner (1961) and Dhrymes, Howrey, Hymans, Kmenta, Leamer, Quandt, Ramsey, Shapiro and Zarnowitz (1972) derive exact procedures to assess whether one additional observation can be predicted using an estimated MLR. Jayatissa (1976) and Jayatissa and Farebrother (1977) extend the latter predictive test to the case of more than one additional observation. However, this test makes use of a non-unique decomposition of the residuals, which has limited its practical appeal. Stewart (1997) proposes an exact procedure to test for structural change using Rao (1973)'s multivariate F-test. This involves the use of a dummy variable to shift the coefficients of the MLR model, where a structural break test procedure would assess whether the shift parameters are zero. The latter hypothesis involves constraints on the augmented

[1] The literature is indeed very vast; see *e.g.* the special issue of the Journal of Econometrics, 1996, or Bai and Perron (1998), Bai and Perron (2003), and the references and surveys cited therein.

[2] Oil price volatility, environmental issues and associated policies and programs generate a lasting interest for energy demand analysis, so surveys appear on a regular basis; see e.g. Bohi (1981), Bohi and Zimmerman (1984), Hawdon (1992), Slade, Kolstad and Weiner (1993) and Madlener (1996). In this context, structural stability is a fundamental issue. There is now ample empirical evidence documenting the implications of parameter nonconstancy for model specification, forecasting and policy simulations; see *e.g.* McAvinchey and Yannopoulos (2003) and the insightful discussion in Pindyck (1999). Undoubtedly, oil shock concerns have fuelled research on structural breaks for more than two decades.

MLR which take the specific uniform linear (UL) form (see Dufour and Khalaf (2002*b*)) and can be tested exactly in some cases using Rao (1973)'s F statistic. Although Stewart's discussion of structural change is limited, the proposed methodology is quite promising particularly for normal MLR models, with known break-points. Note however that not all parameter constancy hypotheses can be put in the form discussed by Stewart (1997). In the event studies literature where break dates are typically known, available procedures are only asymptotic and are known to be unreliable in finite samples, because of dimensionality problems; see e.g. Binder (1985*a*), Binder (1985*b*), Salinger (1992) and Barber and Lyon (1997). Exceptions include Schipper and Thompson (1985)'s applications of the Hotelling T^2 test which requires normal errors. These procedures exploit the same statistical theory presented in Stewart (1997). All other multivariate procedures are asymptotic, even in the absence of dynamic terms in the MLR.

A number of studies [see e.g. Dufour and Khalaf (2002*b*) and the references cited therein] have cast doubt on the reliability of asymptotic MLR-based tests. Problems result from the fact that null distributions typically depend on the error covariance parameters - whose number increases rapidly with the system's dimension. When break dates are unknown, dimensionality problems are compounded with multiple hypothesis concerns. Exact test solutions seem lacking even in univariate contexts [Dufour and Kiviet (1996) and Dufour and Kiviet (1998) are notable exceptions], yet documented size control problems have motivated a number of bootstrap-based procedures [see e.g. Christiano (1992), Dufour and Kiviet (1996), Dufour and Kiviet (1998), Diebold and Chen (1996), Banerjee, Lazarova and Urga (1998), Bekaert, Harvey and Lumsdaine (2002)] which promise improved finite sample properties.

In this paper, we extend the finite-sample justified tests proposed in Dufour and Khalaf (2002*b*) to structural stability hypotheses. These tests allow to assess UL restrictions in MLR models under general parametric distributional assumptions, which include the Gaussian distribution and a wide class of non-gaussian distributions. We consider new and existing predictive and non-predictive break test statistics, and show they can be cast as UL hypothesis tests. This allows to show the tests' location-scale invariance, so that Monte Carlo (MC) test techniques [see Dwass (1957), Barnard (1963), Dufour and Khalaf (2001), Dufour (2002), Dufour and Kiviet (1996), Dufour and Kiviet (1998)] can be applied to obtain provably exact p-values. In a dynamic MLR, extra nuisance parameters are treated as in Dufour and Kiviet (1996), through a two-stage consistent set maximized Monte Carlo (CSMMC) procedure.[3] Our contributions can be classified in five categories.

First, we extend tests for which a finite-sample theory has been supplied for Gaussian distributions (see e.g. Schipper and Thompson (1985) and Stewart (1997)) to the non-Gaussian context.

Second, we define two exact variants of Bai et al. (1998)'s test. To motivate the need to revisit this test, we run a small scale simulation study in a parsimonious non-dynamic MLR and show that Bai et al. (1998)'s test severely over-rejects. We next observe that the sup-type

[3]CSMMC test procedures generally proceed as follows. First, one builds an exact confidence set for the nuisance parameter. Second, the MC p-value for the break test of interest (which depends on the nuisance parameter) is maximized over this confidence set. This procedure may be interpreted as a special form of "maximized MC" (MMC) test [see Dufour (2002)].

statistics [Andrews (1993), Andrews and Ploberger (1994)] for hypotheses studied by Bai et al. (1998) are pivotal when applied to an MLR model; as a result, the MC method can easily be applied to correct their size. We also consider an alternative procedure, based on the product rather than the minimum of individual p-values, to combine the tests over all possible break dates. The method extends the Fisher-Pearson criterion [Fisher (1932), Pearson (1933), Dufour and Khalaf (2002a), Dufour, Khalaf, Bernard and Genest (2004), Dufour, Khalaf and Beaulieu (2003)] to non-independent test statistics.

Third, we consider predictive break test approaches which aim to generalize the tests of Dufour (1980) and Dufour and Kiviet (1996). We show that the multivariate analogue of the Student-t test on time dummies proposed by Dufour (1980) leads to a Hotelling-T^2 type statistic that is F distributed in the normal errors case. We also use the multiple test combination criteria discussed above to define two alternative predictive tests.

Fourth, we extend Wilks (1963)'s multivariate outliers test to the regression context (see Caroni and Prescott (1992), Thode (2002)) and propose exact (non-Bonferonni based) versions of the test.

Fifth, we apply our proposed procedures to the energy demand system estimated by Arsenault et al. (1995). The model consists of a single equation for total energy demand and a set of share equations for energy sources. Both specifications are dynamic and data sets are usually annual, which motivates the use of finite-sample statistical methods.[4]

The paper is organized as follows. In section 2, we set notation and introduce the general statistical framework. Section 3 discusses the various structural break test procedures we consider. Section 4 presents our empirical analysis. We conclude in section 5.

2 Framework

In this section, we introduce the general statistical framework, the models and notations to be used in the paper. Consider the MLR model

$$Y = XB + U, \tag{2.1}$$

where $Y = [Y_1, ..., Y_T]'$ is a $T \times n$ matrix of observations on n dependent variables, X is a $T \times k$ full-column rank regressors matrix, and $U = [U_1, ..., U_T]'$ is the $T \times n$ matrix of error terms. Although our empirical application imposes multivariate normality of U_t, our results are valid under the general distributional assumption used in Dufour and Khalaf (2002b):

$$U_t = JW_t,\ t = 1, ..., T, \tag{2.2}$$

[4] Commonly used structural change procedures in this context remain asymptotic. For instance, Dunstan and Schmidt (1988) use standard Chow tests to assess structural stability of US residential energy demand model for 1971-1974 and 1979-1982 periods; their tests which correct for changing variances do not account for the dynamic nature of the model. McAvinchey and Yannopoulos (2003) approach the problem via forecasting performance using root-mean-square forecast error estimates, and perform standard parameter constancy tests (against level shifts and non-linear threshold effects).

where J is unknown, non-singular and the distribution of the vector $w = vec(W_1, ..., W_T)$ is known. We denote $W = [W_1, ..., W_T]' = U\left(J^{-1}\right)'$ and $\Sigma = JJ'$.[5] It is also convenient to rewrite the model in the following form

$$Y_{it} = X_t' B_i + U_{it}, \;\; i = 1, ..., n, \;\; t = 1, ..., T, \tag{2.3}$$

where Y_{it} are the elements of the matrix Y, X_t' is the row of the matrix X and B_i is the ith column of B. We assume we can condition on X (*i.e.* we can take X as fixed for statistical analysis).

Demand share equations can often be cast in the latter framework. The case we analyze in Section 4 (based on Arsenault et al. (1995)) provides a typical example. We model total energy demand through a two-level system with energy sources substitution, for three sectors of energy use (industrial, commercial and residential). Energy sources common to these three sectors are: fuel oil (O), natural gas (G), electricity (L) and coal (C). For each sector, energy sources market shares are first modelled as a function of their own first lag, and relative prices. At the second level, total energy demand (measured in terajoules) is modelled as a function of its own lag and: real energy price, real income, and a measure of temperature (heating degree days). For each sector (treated as a separate system), the estimated empirical equations are specified as follows. The total demand equation takes the form

$$\ln(\mathrm{TE}_t) = \rho \ln(\mathrm{TE}_{t-1}) + x_t'(\rho)\beta + u_t, \tag{2.4}$$

where $\beta = (a_1, a_2, ..., a_4)'$, $u_1, \ldots, u_T$ are *i.i.d.* $N[0, \sigma^2]$ error terms, $x_t(\rho)$ is the k-dimensional vector $(1, \ln(\mathrm{PE}_t), \ln(\mathrm{IN}_t), (\ln(\mathrm{HDD}_t) - \rho\ln(\mathrm{HDD}_{t-1})))'$, TE_t, PE_t, IN_t and HDD_t denote (respectively) total energy demand (in terajoules), aggregate real price of energy, aggregate real income, and heating degree days for year t.[6] Convergence of the adjustment process requires $0 < \rho < 1$.[7] In this context, market shares equations take the form:

$$\mathrm{MS}_{it} = \lambda\,\mathrm{MS}_{i,t-1} + X_t' B_i + U_{it}, \;\; i = L, O, C, \;\; t = 1, ..., T, \tag{2.5}$$

where $B_L = (b_L, b_{LL}, b_{LO}, b_{LC})'$, $B_O = (b_O, b_{OL}, b_{OO}, b_{OC})'$, $B_C = (b_C, b_{CL}, b_{CO}, b_{CC})'$, MS_{it} and P_{it} denote, respectively, market share and price of energy source $i = L, O, C$, for year t, $X_t = \left(1, \ln\left[\frac{P_{Lt}}{P_{Gt}}\right], \ln\left[\frac{P_{Ot}}{P_{Gt}}\right], \ln\left[\frac{P_{Ct}}{P_{Gt}}\right]\right)'$, and U_{Lt}, U_{Ot}, U_{Ct} are contemporaneously correlated error terms. Adding up-constraints (formally $\mathrm{MS}_{Gt} = 1 - (\mathrm{MS}_{Lt} + \mathrm{MS}_{Ot} + \mathrm{MS}_{Ct})$) imply that λ, which measures the partial adjustment mechanism, must be the same for all equations of the system. Convergence of the adjustment process requires $0 < \lambda < 1$.

Following Dufour and Kiviet (1996), we first derive pivotal statistics (i.e. statistics whose null distribution is known in the sense that it can be simulated) for cases where λ is known (say equal to λ_0). This leads to the transformed model:

$$\mathrm{MS}_{it} - \lambda_0\,\mathrm{MS}_{i,t-1} = X_t' B_i + U_{it}, \;\; i = L, O, C, \;\; t = 1, ..., T, \tag{2.6}$$

[5] In the normal error case, Σ is the covariance matrix of U_t.

[6] To ensure that temperature effects for year t are restricted to year t, the coefficient of HDD_{t-1} is set to $-\rho a_4$. Temperature effects are excluded from the demand equation for industrial sector.

[7] In section 4, we apply Dufour and Kiviet (1996)'s tests (which we summarize in Appendix A) to the latter equation.

which is a special case of the MLR (2.1)-(2.3). Applying the MC test procedure to the proposed pivots leads to exact p-values, conditional on λ. To account for an unknown λ, we maximize the latter p-values over a consistent nuisance parameter set estimate (of level $1-\alpha_1$). If the maximized p-value is referred to level α_2, then the overall level of the two-step test is $\alpha_1+\alpha_2$.

3 Multivariate Structural Change Tests

In this section, we propose three finite-sample justified test procedures for structural change applicable to the MLR model (2.1). We consider two exact versions of Bai et al. (1998)'s sup-F test, as well as new predictive tests which extend the methodology of Dufour (1980) to the multivariate context.

3.1 Exact Versions of Bai, Lumsdaine and Stock (1998)'s Test

Following the general set-up of Bai et al. (1998), consider model (2.1) augmented as follows:

$$Y = XB + D_s\Delta_s + U = Z_s\Theta_s + U, \qquad Z_s = \left[\begin{array}{cc} X & D_s \end{array}\right], \;\; \Theta_s = \left[\begin{array}{c} B \\ \Delta_s \end{array}\right], \tag{3.7}$$

where $s \in [T_*+1 : T-T_*]$ corresponds to a break date, T_* is the trimming parameter, D_s is a matrix with typical row equal to $D_{ts}\overline{X}_t'$, D_{ts} is the dummy variable

$$\begin{aligned} D_{ts} &= 1, \quad t > s, \\ &= 0, \quad t \leq s, \end{aligned}$$

$\overline{X}_t'$ is the row of $\overline{X} = XQ_X$ and Q_X is a $K \times q_X$ selection matrix (of zeros and ones) which allows to specify the regressors whose coefficients are tested for constancy.

3.1.1 Finite sample theory

Structural stability corresponds to null hypotheses

$$H_{0s}^* : \Delta_s = 0 \Leftrightarrow R^*\Theta_s = 0, \quad R^* = \left[\begin{array}{cc} \mathbf{0}_{q_X\times K} & I_{q_X} \end{array}\right] \tag{3.8}$$

where $\mathbf{0}_{l\times m}$ denotes an $l \times m$ matrix of zeros, and

$$H_0^* \Longleftrightarrow \cap_{s\,\in\,[T_*+1\,:\,T-T_*]}\,(H_{0s}^*)\,. \tag{3.9}$$

In this context, we next derive the exact null distribution of the commonly used test statistics and propose alternative procedures to test H_0^*.

Theorem 1 *Under* (2.1), (2.2)*, the test statistics associated with* (3.8)-(3.9)

$$\Lambda^* = \sup_{s \,\in\, [T_*+1\,:\,T-T_*]} \{-T\ln(\Lambda_s^*)\}, \tag{3.10}$$

$$\Lambda_s^* = |\mathcal{S}_s^*| / |\mathcal{S}^0|, \tag{3.11}$$

$$\mathcal{F}^* = \sup_{s \,\in\, [T_*+1\,:\,T-T_*]} \{\mathcal{F}_s^*\}, \tag{3.12}$$

$$\mathcal{F}_s^* = T\,\mathrm{trace}\left((\mathcal{S}_s^*)^{-1}\left(\mathcal{S}^0 - \mathcal{S}_s^*\right)\right), \tag{3.13}$$

$$\mathcal{S}_s^* = \widehat{U}_s^{*\prime}\widehat{U}_s^*, \qquad \mathcal{S}^0 = \widehat{U}^{0\prime}\widehat{U}^0, \tag{3.14}$$

$$\mathrm{LP}(\Lambda^*) = -2\sum_{s \,\in\, [T_*+1\,:\,T-T_*]} \ln(pv\,[\Lambda_s^*]), \tag{3.15}$$

$$pv\,[\Lambda_s^*] = G_F\left(\left[\frac{1-(\Lambda_s^*)^{1/m_2^*}}{(\Lambda_s^*)^{1/m_2^*}}\frac{m_1^*m_2^*-2m_3^*}{nq_X}\right] \mid nq_X,\, m_1^*m_2^*-2m_3^*\right), \tag{3.16}$$

$$m_1^* = T-(K+q_X)-\frac{n-q_X+1}{2},\ m_2^* = \sqrt{\frac{n^2q_X^2-4}{n^2+q_X^2-5}},\ m_3^* = \frac{nq_X-2}{4},$$

where $\widehat{U}^0$ *and* $\widehat{U}_s^*$ *are the OLS residuals from* (2.1) *and* (3.7) *respectively, and* $G_F(x|\nu_1,\nu_2)$ *denotes the survival function, evaluated at point* x*, of the F distribution with* (ν_1,ν_2) *degrees of freedom, are distributed (under the null hypothesis) following the pivotal quantities respectively:*

$$\overline{\Lambda^*} = \sup_{s \,\in\, [T_*+1\,:\,T-T_*]} \left\{-T\ln\left(\overline{\Lambda_s^*}\right)\right\}, \tag{3.17}$$

$$\overline{\Lambda_s^*} = \frac{\left|W'\left(I-Z_s(Z_s'Z_s)^{-1}Z_s'\right)W\right|}{\left|W'\left(I-X(X'X)^{-1}X'\right)W\right|}, \tag{3.18}$$

$$\overline{\mathcal{F}^*} = \sup_{s \,\in\, [T_*+1\,:\,T-T_*]} \left\{\overline{\mathcal{F}_s^*}\right\}, \tag{3.19}$$

$$\overline{\mathcal{F}_s^*} = T\,\mathrm{trace}\left(\left(\overline{\mathcal{S}_s^*}\right)^{-1}\left(\overline{\mathcal{S}^0}-\overline{\mathcal{S}_s^*}\right)\right), \tag{3.20}$$

$$\overline{\mathcal{S}_s^*} = W'\left(I-Z_s\left(Z_s'Z_s\right)^{-1}Z_s'\right)W, \qquad \overline{\mathcal{S}^0} = W'\left(I-X\left(X'X\right)^{-1}X'\right)W \tag{3.21}$$

$$\mathrm{LP}\left(\overline{\Lambda^*}\right) = -2\sum_{s \,\in\, [T_*+1\,:\,T-T_*]} \ln\left(pv\left[\overline{\Lambda_s^*}\right]\right), \tag{3.22}$$

$$pv\left[\overline{\Lambda_s^*}\right] = G_F\left(\left[\frac{1-\left(\overline{\Lambda_s^*}\right)^{1/m_2^*}}{\left(\overline{\Lambda_s^*}\right)^{1/m_2^*}}\frac{m_1^*m_2^*-2m_3^*}{nq_X}\right] \mid nq_X,\, m_1^*m_2^*-2m_3^*\right), \tag{3.23}$$

where W *is defined by* (2.2).

Proof. Following the derivations from Dufour and Khalaf (2002*b*, section (3)), we can write Λ_s^* under the null hypothesis as follows:

$$\Lambda_s^* = \frac{\left|U'\left(I - Z_s\left(Z_s'Z_s\right)^{-1}Z_s'\right)U\right|}{\left|U'\left(I - X\left(X'X\right)^{-1}X'\right)U\right|} = \frac{\left|W'\left(I - Z_s\left(Z_s'Z_s\right)^{-1}Z_s'\right)W\right|}{\left|W'\left(I - X\left(X'X\right)^{-1}X'\right)W\right|}. \tag{3.24}$$

The latter obtains on observing that $W = U(J^{-1})'$. The same observation implies that

$$\begin{aligned}
\left(\mathcal{S}_s^*\right)^{-1}\left(\mathcal{S}^0 - \mathcal{S}_s^*\right) &= \left(JJ^{-1}\mathcal{S}_s^*(J^{-1})'J'\right)^{-1}\left(JJ^{-1}\mathcal{S}^0(J^{-1})'J' - JJ^{-1}\mathcal{S}_s^*(J^{-1})'J'\right) \\
&= \left(J^{-1}\right)'\left(J^{-1}\mathcal{S}_s^*(J^{-1})'\right)^{-1}J^{-1}J\left(J^{-1}\mathcal{S}^0(J^{-1})' - J^{-1}\mathcal{S}_s^*(J^{-1})'\right)J' \\
&= \left(J^{-1}\right)'\left(\overline{\mathcal{S}_s^*}\right)^{-1}\left(\overline{\mathcal{S}^0} - \overline{\mathcal{S}_s^*}\right)J'.
\end{aligned}$$

Thus

$$\begin{aligned}
\text{trace}\left(\left(\mathcal{S}_s^*\right)^{-1}\left(\mathcal{S}^0 - \mathcal{S}_s^*\right)\right) &= \text{trace}\left(\left(J^{-1}\right)'\left(\overline{\mathcal{S}_s^*}\right)^{-1}\left(\overline{\mathcal{S}^0} - \overline{\mathcal{S}_s^*}\right)J'\right), \\
&= \text{trace}\left(\left(J^{-1}\right)'J'\left(\overline{\mathcal{S}_s^*}\right)^{-1}\left(\overline{\mathcal{S}^0} - \overline{\mathcal{S}_s^*}\right)\right), \\
&= \text{trace}\left(\left(\overline{\mathcal{S}_s^*}\right)^{-1}\left(\overline{\mathcal{S}^0} - \overline{\mathcal{S}_s^*}\right)\right).
\end{aligned}$$

■

Λ^* and $\mathcal{F}^*$ correspond to the standard sup-QLR and sup-Wald statistics. Indeed, Λ_s^* is a monotonic transformation of the standard Gaussian QLR statistic associated with H_{0s}^*, which may be calculated in our context as

$$-T\ln\left(\Lambda_s^*\right) = -T\sum_{i=1}^{n}\ln(1 - \widehat{\mu}_i)$$

where $\widehat{\mu}_1 \geq \widehat{\mu}_2 \geq ... \geq \widehat{\mu}_n$ are the eigen values of

$$\left(\widehat{U}^{0\prime}\widehat{U}^0\right)^{-1}\left(\widehat{U}^{0\prime}\widehat{U}^0 - \widehat{U}_s^{*\prime}\widehat{U}_s^*\right).$$

$\mathcal{F}_s^*$ is the standard Wald statistic used by Bai et al. (1998) associated with H_{0s}^* (see Appendix B for a proof, Berndt and Savin (1977) and Gouriéroux, Monfort and Renault (1995)); the latter can also be written in terms of $\widehat{\mu}_1 \geq \widehat{\mu}_2 \geq ... \geq \widehat{\mu}_n$ as follows

$$\mathcal{F}_s^* = T\sum_{i=1}^{n}\frac{\widehat{\mu}_i}{1 - \widehat{\mu}_i}.$$

LP (Λ^*) is an alternative statistic we propose for the unknown break case, based on the Fisher-Pearson multiple hypothesis test criterion [Fisher (1932), Pearson (1933), Dufour and Khalaf

(2002a), Dufour et al. (2004)] and the following approximation to the null distribution of Λ_s^* (see Rao (1973), Stewart (1997) and Dufour and Khalaf (2002b)):

$$\frac{1-(\Lambda_s^*)^{1/m_2^*}}{(\Lambda_s^*)^{1/m_2^*}}\frac{m_1^*m_2^*-2m_3^*}{nq_X}\sim F(nq_X, m_1^*m_2^*-2m_3^*). \tag{3.25}$$

In other words, $pv\,[\Lambda_s^*]$ are p-values based on (3.25) for Λ_s^*. Though the latter p-values are exact only under normal errors when $\min(n,\,q_X)\leq 2$, the combined criterion LP (Λ^*) is pivotal under the conditions of Theorem 1.

Indeed, Theorem 1 shows that the null distributions of the statistics (3.10) - - (3.15) do not depend on B, Δ nor J (and thus not on Σ) and may easily be simulated if draws from the distribution of $(W_1,\ \ldots\ ,\ W_T)$ are available. This entails that a Monte Carlo exact test procedure may be easily applied based on these statistics to obtain exact p-values. The procedure may be summarized as follows (for further details, see Dufour (2002)).

Using N draws from the distribution of W and (3.17) - - (3.23), obtain N simulated values of the test statistics. These provide N realizations from the null distribution of the tests statistics, under the conditions of Theorem 1. Then derive the (empirical) p-values

$$\hat{p}_N\,(.)=\frac{N\hat{G}_N\,(.)+1}{N+1}, \tag{3.26}$$

where $N\hat{G}_N\,(.)$ is the number of simulated values greater than or equal to the observed value of the test statistics. The MC tests defined by the critical regions $\hat{p}_N\,(.)\leq\alpha$ will have size α exactly (for finite N and T), given the invariance results of Theorem 1, despite the non-standard statistics considered.

One may wish to test parameter constancy in a subset of equations; this is also possible in our framework through a null hypothesis of the form $H_{0s}^*: R^*\Theta Q_Y=0$, where Q_Y is an $n\times q_Y$ selection matrix Q_Y (of zeros and ones) which allows to specify the equations of interest. The test problem amounts to assessing $R^*\Theta_{Q_Y}=0$ in the context of the reparametrized MLR:

$$Y_{Q_Y}=Z_s\Theta_{Q_Y}+U_{Q_Y} \tag{3.27}$$

where $Y_{Q_Y}=YQ_Y$, $\Theta_{Q_Y}=\Theta Q_Y$ and $U_{Q_Y}=UQ_Y$. The analysis and distributional results then proceed as above.

3.1.2 Size and power study

To motivate the need for finite-sample justified break tests, we have conducted a small scale simulation study to illustrate the properties of Bai et al. (1998)'s asymptotic test and its exact counterparts proposed here.[8] The model we consider is a special case of (3.7) where $n=1,3,10$, $T=25,40,80$. We use two designs for the regressor matrix X; both include two

[8]For a more extensive simulation study, see *Clément Yelou*, "Tests valides en échantillons finis pour les régressions multivariées et les modèles de choix, avec applications en économie de l'énergie et du transport", PhD Thesis, 2004, Université Laval.

Table 1. Empirical Size of sup-type tests: trend regressor

Nominal size: 5%	$n=1$		$n=2$		$n=5$		$n=10$		$n=20$	
T	Asy.	Λ^*	Asy.	Λ^*	Asy.	Λ^*	Asy.	Λ^*	Asy.	Λ^*
20	19.2	4.4	32.1	4.1	83.5	6.3	99.9	5.8	-	-
30	15.7	5.3	20.3	4.8	54.2	3.8	96.0	4.7	100	4.3
35	10.7	3.8	16.9	3.9	46.9	5.2	91.8	6.0	99.1	5.4
40	11.2	5.0	16.2	5.7	35.8	4.0	83.3	4.7	98.4	6.4
50	10.1	5.5	13.1	5.3	29.0	4.5	68.2	4.7	89.9	4.8
60	10.4	4.8	10.0	4.1	22.6	5.4	51.9	4.5	79.9	5.7
80	8.5	5.4	11.0	5.4	19.5	4.3	36.4	4.2	57.0	4.3
100	7.6	5.0	9.6	4.5	16.4	5.1	30.4	5.4	43.3	5.9
120	7.8	4.9	11.2	6.0	14.3	4.3	23.6	4.9	37.0	4.7
140	8.0	5.4	8.9	5.1	13.3	5.8	19.5	4.2	27.9	5.5
180	6.9	4.1	7.5	4.3	11.3	4.9	17.8	5.5	23.0	6.1

Note: Numbers reported are empirical rejection rates under the null hypothesis of structural stability for the asymptotic Bai et al. (1998)'s test [in the column titled ASY], and for the exact MC test based on the sup-type test as defined in (3.10) [in the column titled Λ^*]; n is the number of equations in the system; the system includes two regressors: the constant term and a time trend. See section 3.1.2 for details concerning the simulation design.

regressors, namely the constant term and: (i) a time trend for the first design, (ii) a standard normal variate for the second design (drawn only once). The power study focuses on a single design which includes all three regressors (the constant, trend and normal variate). We focus on the case where a break may occur in the regression intercept, so $Q_X = (1,0,0)'$. $B = \left[(0,1,1)', ..., (0,1,1)'\right]$ and $\Delta = \xi_0 \iota_n'$ under the alternative hypothesis where $\iota_n = (1, ..., 1)'$ and ξ_0 is a parameter which controls the magnitude of the break; the values considered are $\xi_0 = 0.25; 0.5; 0.75; 1.5; 2; 5; 10$. The regression errors are drawn as standard multivariate normal, so that $\Sigma = I_n$ in all experiments. We consider a one time break at dates $T_0 = [.5T] + 1, [.85T], [.95T]$, where $[.]$ is the integer part function. We use 1000 replications and $N = 99$ in the MC tests. The trimming parameter is set to .10. For $n = 20$, due to the limitations of Andrews (1993) and Andrews (2003)'s table for asymptotic critical values, only systems with one regressor are considered, the constant term. We study Bai et al. (1998)'s test and our QLR-based variants. Our results are presented in tables 1, 2 and 3.

As may be seen from tables 1 and 2, our proposed MC tests achieve size control in all cases. Thus, the size of the sample or the system's dimensionality do not cause any problem for the exact MC tests we propose. In contrast, Bai et al. (1998)'s asymptotic test suffers from severe over-rejections, even in this very simple normal model. Note that: (i) size distortions worsen the higher the dimension of the system; (ii) size problems occur more markedly for the design with a trend regressor. The power study confirms the good performance of the MC tests. Indeed, the latter display good power, for any dimension even with small sample sizes.

Table 2. Empirical size of sup-type tests: normal regressor

Nominal size: 5%	$n=1$		$n=2$		$n=5$		$n=10$	
T	Asy.	Λ^*	Asy.	Λ^*	Asy.	Λ^*	Asy.	Λ^*
20	13.8	4.1	29.9	4.7	80.7	5.3	100	4.7
30	9.9	5.5	15.4	4.1	46.1	4.6	95.7	5.7
35	10.3	5.8	15.1	5.3	40.7	5.3	88.0	5.6
40	10.2	5.9	12.2	5.5	30.7	5.2	76.1	4.7
50	5.8	3.7	9.1	4.9	22.8	4.2	58.6	4.2
60	6.3	4.7	8.8	5.2	17.3	4.8	44.9	5.0
80	5.8	5.5	8.5	6.0	14.3	4.6	31.4	6.2
100	5.1	5.0	6.1	4.8	9.8	4.2	25.6	6.1
120	5.7	5.1	8.1	6.6	10.4	6.2	19.0	5.0
140	5.3	4.7	6.8	5.4	8.3	5.3	16.0	5.4
180	4.2	4.0	5.5	5.6	7.4	4.5	13.3	5.7

Note: Numbers reported are empirical rejection rates under the null hypothesis of structural stability for the asymptotic Bai et al. (1998)'s test [in the column titled ASY], and for the exact MC test based on the sup-type test as defined in (3.10) [in the column titled Λ^*]; n is the number of equations in the system; the system includes two regressors: the constant term and a standard normal variate. See section 3.1.2 for details concerning the simulation design.

3.2 Predictive Test Procedures

We now describe the predictive test approach, which may be seen as a multivariate generalization of the univariate break tests underlying Dufour and Kiviet (1996)'s procedures. This approach assumes that the model is stable over a known subsample, say $[1 : T_1]$. Let $T_2 = T - T_1$. Consider

$$Y_{it} = X_t' B_i + \sum_{s=T_1+1}^{T} d_{ts}\gamma_{is} + U_{it}, \quad i = 1, ..., n, \quad t = 1, ..., T, \tag{3.28}$$

$$d_{ts} = 1, \quad t = s, \qquad = 0, \quad t \neq s. \tag{3.29}$$

which obtains from (2.3) through the addition, in each equation, of the time-dummy variables d_{ts}. The latter can be written in matrix form as

$$Y = XB + \overline{D}\Gamma + U = Z\Pi + U \tag{3.30}$$

$$\overline{D} = \begin{bmatrix} 0_{T_1 \times T_2} \\ I_{T_2} \end{bmatrix}, \; Z = \begin{bmatrix} X & \overline{D} \end{bmatrix} = \begin{bmatrix} X_{(1)} & 0 \\ X_{(2)} & I_{T_2} \end{bmatrix}, \; \Pi = \begin{bmatrix} B \\ \Gamma \end{bmatrix},$$

where the partitioning of X into $X_{(1)}$ and $X_{(2)}$ refers to the regressor matrices associated with each subsample, $[1 : T_1]$ and $[T_1 + 1 : T]$. Let us partition the matrices Y, U and W [from 2.2] into $Y_{(1)}$ and $Y_{(2)}$, $U_{(1)}$ and $U_{(2)}$, and $W_{(1)}$ and $W_{(2)}$, conforming to the partionning of

Table 3. Empirical power of structural stability tests

		ξ_0	1.5		5		10	
T	n	T_0	Λ^*	LP (Λ^*)	Λ^*	LP (Λ^*)	Λ^*	LP (Λ^*)
25	1	[.5T]+1	10.5	5.9	96.4	24.0	100.0	77.3
		[.85T]	20.0	25.0	99.9	98.5	100.0	100.0
		[.95T]	8.1	7.0	75.8	32.0	100.0	90.8
	3	[.5T]+1	14.0	8.4	100.0	26.6	100.0	64.6
		[.85T]	32.9	36.8	100.0	99.9	100.0	100.0
		[.95T]	10.2	7.2	95.8	46.3	100.0	96.5
	10	[.5T]+1	14.0	8.5	98.4	19.6	100.0	31.2
		[.85T]	27.5	30.9	99.9	80.8	100.0	94.1
		[.95T]	11.3	7.5	89.6	29.7	100.0	65.9
40	1	[.5T]+1	21.1	11.8	100.0	84.3	100.0	100.0
		[.85T]	37.1	42.8	100.0	100.0	100.0	100.0
		[.95T]	11.9	12.1	91.2	69.6	100.0	99.4
	3	[.5T]+1	39.2	16.2	100.0	92.4	100.0	100.0
		[.75T]	66.2	67.7	100.0	100.0	100.0	100.0
		[.90T]	19.5	17.0	98.9	79.6	100.0	99.0
	10	[.5T]+1	59.4	19.4	100.0	69.6	100.0	92.7
		[.75T]	85.8	80.4	100.0	100.0	100.0	100.0
		[.90T]	25.4	18.6	96.3	54.8	99.9	64.9
80	1	[.5T]+1	48.2	26.5	100.0	100.0	100.0	100.0
		[.75T]	68.5	72.8	100.0	100.0	100.0	100.0
		[.90T]	23.3	24.4	100.0	99.6	100.0	100.0
	3	[.5T]+1	88.2	47.8	100.0	100.0	100.0	100.0
		[.85T]	97.2	97.4	100.0	100.0	100.0	100.0
		[.95T]	48.9	40.4	100.0	100.0	100.0	100.0
	10	[.5T]+1	99.9	68.9	100.0	100.0	100.0	100.0
		[.85T]	100.0	100.0	100.0	100.0	100.0	100.0
		[.95T]	74.9	56.3	100.0	100.0	100.0	100.0

Note: Numbers reported are empirical rejection rates under the alternative hypothesis for the MC tests defined in Theorem 1. The break date is T_0, where [.] is the integer part function. $\xi_0 = 1.5; 5; 10$ is the jump size. The tests assume an unknown break date. See section 3.1.2 for details concerning the simulation design.

X. Let $\hat{U}_{(1,s)}$ denote the OLS residual matrix from the regression of Y on X using the first T_1 observations and the observations at date s. Let $Y_{(1,s)}$, $X_{(1,s)}$, $U_{(1,s)}$ and $W_{(1,s)}$ refer to elements of Y, X, U and W corresponding to the first T_1 observations to which we append the corresponding elements for date s. Formally,

$$Y_{(1,s)} = \begin{bmatrix} Y_{(1)} \\ Y_s' \end{bmatrix}, \quad X_{(1,s)} = \begin{bmatrix} X_{(1)} \\ X_s' \end{bmatrix}, \quad U_{(1,s)} = \begin{bmatrix} U_{(1)} \\ U_s' \end{bmatrix}, \quad W_{(1,s)} = \begin{bmatrix} W_{(1)} \\ W_s' \end{bmatrix} \tag{3.31}$$

for $s = T_1 + 1, ..., T$, where Y_s', X_s', U_s' and W_s' refer to the s-th row of the matrices Y, X, U and W respectively.

In this context, the null hypothesis of structural stability takes the form

$$H_0 : R\Pi = 0, \quad R = \begin{bmatrix} 0_{T_2 \times K} & I_{T_2} \end{bmatrix}. \tag{3.32}$$

In addition, the null hypothesis of no structural change at a given observation s beyond the stable period, $s = T_1 + 1, ..., T$, can be written as

$$H_{0s} : R_s\Pi = 0, \quad s \in [T_1 + 1 \ : \ T], \tag{3.33}$$

where R_s is the row vector which corresponds to the sth row of the matrix R defined in (3.32). The latter hypothesis is useful in order to date structural breaks. Both (3.32)-(3.33) are UL.

Theorem 2 *Under* (3.30), (2.2), *and the notation from* (3.31), *the test statistics associated with* (3.32)-(3.33)*:*

$$\Lambda = \frac{\left|\widehat{U}'\widehat{U}\right|}{\left|\widehat{U}^{0\prime}\widehat{U}^{0}\right|}, \tag{3.34}$$

$$\Lambda_s = \frac{\left|\hat{U}'\hat{U}\right|}{\left|\hat{U}_s^{0\prime}\hat{U}_s^{0}\right|}, \tag{3.35}$$

$$\Lambda_{\max} = \sup_{s \,\in\, \mathcal{J}_{[T_1+1\,:\,T]}} \{-T\ln(\Lambda_s)\}, \tag{3.36}$$

$$\mathrm{LP}(\Lambda) = -2 \sum_{s \,\in\, \mathcal{J}_{[T_1+1\,:\,T]}} \ln\left(pv\left[\Lambda_s\right]\right), \tag{3.37}$$

$$pv\left[\Lambda_s\right] = G_F\left(\frac{1-\Lambda_s}{\Lambda_s}\frac{T_1-(K-1)-n}{n} \mid n,\ T_1-(K-1)-n\right) \tag{3.38}$$

where $\widehat{U}^0$ *and* $\widehat{U}$ *are the OLS residuals from* (2.1) *and* (3.30) *respectively,* $\widehat{U}_s^0$ *is the constrained* (*imposing* (3.33)) *OLS residual from* (3.30)*,* $\mathcal{J}_{[T_1+1\,:\,T]}$ *is any subset of dates of interest in* $[T_1 + 1 \ : \ T]$*, and* $G_F(x|\nu_1, \nu_2)$ *denotes the survival function, evaluated at point* x*, of the F distribution with* (ν_1, ν_2) *degrees of freedom, are distributed (under the null hypothesis) following* *the pivotal quantities respectively:*

$$\overline{\Lambda} = \frac{\left|W'\left(I - Z\left(Z'Z\right)^{-1} Z'\right) W\right|}{\left|W'\left(I - X\left(X'X\right)^{-1} X'\right) W\right|} \tag{3.39}$$

$$= \frac{\left|W'_{(1)}\left(I - X_{(1)}\left(X'_{(1)}X_{(1)}\right)^{-1} X_{(1)}\right) W_{(1)}\right|}{\left|W'\left(I - X\left(X'X\right)^{-1} X'\right) W\right|} \tag{3.40}$$

$$\overline{\Lambda}_s = \frac{\left|W'\left(I - Z\left(Z'Z\right)^{-1} Z'\right) W\right|}{\left|W'M_{(s)}W\right|} \tag{3.41}$$

$$= \frac{\left|W'_{(1)}\left(I - X_{(1)}\left(X'_{(1)}X_{(1)}\right)^{-1} X_{(1)}\right) W_{(1)}\right|}{\left|W'_{(1,s)}\left(I - X_{(1,s)}\left(X'_{(1,s)}X_{(1,s)}\right)^{-1} X'_{(1,s)}\right) W'_{(1,s)}\right|} \tag{3.42}$$

$$\overline{\Lambda}_{\max} = \sup_{s \,\in\, \mathcal{J}_{[T_1+1\,:\ \ T]}} \left\{-T\ln(\overline{\Lambda}_s)\right\}, \tag{3.43}$$

$$\mathrm{LP}\left(\overline{\Lambda}\right) = -2 \sum_{s \,\in\, \mathcal{J}_{[T_1+1\,:\ \ T]}} \ln\left(pv\left[\overline{\Lambda}_s\right]\right), \tag{3.44}$$

$$pv\left[\overline{\Lambda}_s\right] = G_F\left(\frac{1-\overline{\Lambda}_s}{\overline{\Lambda}_s}\frac{T_1-(K-1)-n}{n} \mid n,\ T_1-(K-1)-n\right) \tag{3.45}$$

where $M_{(s)} = \left(I - Z\left(Z'Z\right)^{-1} Z'\right) + Z\left(Z'Z\right)^{-1} R'_s\left[R_s\left(Z'Z\right)^{-1} R'_s\right]^{-1} R_s\left(Z'Z\right)^{-1} Z'$, *and* W *is defined by* (2.2).

Proof. Following the arguments of Theorem 1, under the null hypothesis:

$$\Lambda = \frac{\left|U'\left(I - Z\left(Z'Z\right)^{-1} Z'\right) U\right|}{\left|U'\left(I - X\left(X'X\right)^{-1} X'\right) U\right|} = \frac{\left|W'\left(I - Z\left(Z'Z\right)^{-1} Z'\right) W\right|}{\left|W'\left(I - X\left(X'X\right)^{-1} X'\right) W\right|},$$

$$\Lambda_s = \frac{\left|U'\left(I - Z\left(Z'Z\right)^{-1} Z'\right) U\right|}{\left|U'M_{(s)}U\right|} = \frac{\left|W'\left(I - Z\left(Z'Z\right)^{-1} Z'\right) W\right|}{\left|W'M_{(s)}W\right|}.$$

Applying the (univariate) results of Dufour (1980) to each equation of the MLR under consideration implies that: (i) the OLS estimate of B in (3.30) may be obtained by the regression of $Y_{(1)}$ on $X_{(1)}$, (ii) the OLS estimate $\hat{\Gamma}$ of Γ obtains as $Y_{(2)} - X_{(2)}\left(X'_{(1)}X_{(1)}\right)^{-1} X_{(1)}Y_{(1)}$. [9] It

[9] This means that $\hat{\Gamma}$ measures "the deviations" of the observed endogenous variables $Y_{(2)}$ from their predicted values $X_{(2)}\left(X'_{(1)}X_{(1)}\right)^{-1} X_{(1)}Y_{(1)}$ assuming parameter constancy. In event studies models, the latter interpretation leads to the definition of *abnormal returns* [Brown and Warner (1980), Binder (1985*a*), Brown and Warner (1985)].

follows that

$$\begin{aligned}
\Lambda &= \frac{\left|U_{(1)}'\left(I - X_{(1)}\left(X_{(1)}'X_{(1)}\right)^{-1}X_{(1)}\right)U_{(1)}\right|}{\left|U'\left(I - X\left(X'X\right)^{-1}X'\right)U\right|} \\
&= \frac{\left|W_{(1)}'\left(I - X_{(1)}\left(X_{(1)}'X_{(1)}\right)^{-1}X_{(1)}\right)W_{(1)}\right|}{\left|W'\left(I - X\left(X'X\right)^{-1}X'\right)W\right|}, \\
\Lambda_s &= \frac{\left|U_{(1)}'\left(I - X_{(1)}\left(X_{(1)}'X_{(1)}\right)^{-1}X_{(1)}\right)U_{(1)}\right|}{\left|U_{(1,s)}'\left(I - X_{(1,s)}\left(X_{(1,s)}'X_{(1,s)}\right)^{-1}X_{(1,s)}'\right)U_{(1,s)}\right|} \\
&= \frac{\left|W_{(1)}'\left(I - X_{(1)}\left(X_{(1)}'X_{(1)}\right)^{-1}X_{(1)}\right)W_{(1)}\right|}{\left|W_{(1,s)}'\left(I - X_{(1,s)}\left(X_{(1,s)}'X_{(1,s)}\right)^{-1}X_{(1,s)}'\right)W_{(1,s)}'\right|},
\end{aligned}$$

on observing that $\widehat{U}'\widehat{U} = U_{(1)}'\left(I - X_{(1)}\left(X_{(1)}'X_{(1)}\right)^{-1}X_{(1)}\right)U_{(1)}$ and $\hat{U}_s^{0\prime}\hat{U}_s^0 = \hat{U}_{(1,s)}'\hat{U}_{(1,s)}$. ■

Under normal errors,

$$\frac{1-\Lambda_s}{\Lambda_s}\frac{T_1-(K-1)-n}{n} \sim \mathrm{F}\left(n,\ T_1-(K-1)-n\right),$$

which holds exactly, applying Rao (1973)'s distributional result (see Stewart (1997) and Dufour and Khalaf (2002*b*)); for this special case, a monotonic transformation of Λ_s yields Hotelling's T^2 test. This is the statistic proposed by Hughes and Ricks (1984), Binder (1985*b*) and Schipper and Thompson (1985) for event studies. The statistic Λ is also used in the latter literature, where Rao (1973)'s results yield the following approximate null distribution:

$$\frac{1-\Lambda^{1/m_2}}{\Lambda^{1/m_2}}\frac{m_1m_2-2m_3}{nT_2} \sim F(nT_2, m_1m_2-2m_3)$$

$$m_1 = T-(K+T_2)-\frac{n-T_2+1}{2},\ m_2 = \sqrt{\frac{n^2T_2^2-4}{n^2+T_2^2-5}},\ m_3 = \frac{nT_2-2}{4}.$$

The latter holds exactly under normal errors when $\min(n, T_2) \leq 2$.

Our results extend beyond available ones from the event studies literature, in several ways. Indeed, normality is not necessary to our exact testing framework: Theorem 2 shows that given the more general assumption (2.2), the null distributions of Λ, Λ_s, $\Lambda_{\max}$ and $\mathrm{LP}(\Lambda)$ do not depend on B, Γ nor J (and thus not on Σ) and may easily be simulated (to obtain exact MC p-values as described above) if draws from the distribution of $(W_1, \ldots, W_T)$ are available. We also emphasize that $\Lambda_{\max}$ and $\mathrm{LP}(\Lambda)$ (which may be seen as alternatives to the joint F test) have not been studied to date. The flexibility of the MC method allows to go beyond standard procedure, in the construction of valid predictive break tests. Finally, it is interesting to note that the statistics Λ_s and $\Lambda_{\max}$ are related to the ones proposed by Wilks (1963) for the detection of multivariate outliers (see Caroni and Prescott (1992), and Thode (2002))) in location-scale models. We consider these statistics in the next section and propose extensions to the MLR models with covariates.

3.3 An Adaptation of Wilks' Multivariate Outlier Test

Consider the augmented regression

$$Y_{it} = X_t' B_i + d_{ts}\gamma_{is} + U_{it}, \quad i = 1, ..., n, \ , \quad t = 1, ..., T, \quad s \in [1, ..., T] \, , \tag{3.46}$$

where d_{ts} is the dummy variable defined in (3.29); in contrast with (3.28), (3.46) includes an exclusion dummy for time s only. The latter can be written in matrix form as

$$\begin{aligned} Y &= XB + d_s\gamma_s + U = X_{(d_s)}\Pi_s + U \qquad (3.47)\\ X_{(d_s)} &= \begin{bmatrix} X & d_s \end{bmatrix}, \ \Pi_s = \begin{bmatrix} B_1 & \cdots & B_n \\ \gamma_{1s} & \cdots & \gamma_{ns} \end{bmatrix}, d_s = (d_{1s}, ..., d_{ts}, ..., d_{Ts})'. \end{aligned}$$

Observe that the OLS residuals from (3.46) correspond to the OLS residuals from regression (2.1), applied to the whole sample excluding observation s. In this context, we consider

$$H_0^{**} \iff \cap_{s \, \in \, [1 \, : \, T]} \left(H_{0s}^{**} \right), \tag{3.48}$$

$$H_{0s}^{**} \quad : \quad R_s^{**}\Pi_s = 0, \tag{3.49}$$

where R_s^{**} is the $(K+1)$-dimensional row vector with all elements equal to zero except the last one which is 1.

Theorem 3 *Under* (3.47) *and* (2.2), *the test statistics associated with* (3.48)-(3.49)*:*

$$\Lambda_s^{**} = \frac{\left| \widehat{U}_s^{**\prime}\widehat{U}_s^{**} \right|}{\left| \widehat{U}^{0\prime}\widehat{U}^{0} \right|}, \tag{3.50}$$

$$\Lambda_{\max}^{**} = \sup_{s \in \ \mathcal{J}_{[1 \, : \, T]}} \left\{ -T \ln\left(\Lambda_s^{**}\right) \right\}, \tag{3.51}$$

$$\text{LP}\left(\Lambda^{**}\right) = -2 \sum_{s \in \ \mathcal{J}_{[1 \, : \, T]}} \ln\left(pv\left[\Lambda_s^{**}\right]\right), \tag{3.52}$$

$$pv\left[\Lambda_s^{**}\right] = G_F\left(\frac{1-\Lambda_s^{**}}{\Lambda_s^{**}} \frac{T-K-n}{n} \mid n, \ T-K-n \right) \tag{3.53}$$

where $\widehat{U}^0$ *and* $\widehat{U}_s^{**}$ *are the OLS residuals from* (2.1) *and* (3.46), $\mathcal{J}_{[1 \, : \, T]}$ *is any subset of dates of interest in* $[1 \ : \ T]$, *and* $G_F\left(x|\nu_1, \nu_2\right)$ *denotes the survival function, evaluated at point* x, *of* *the F distribution with (ν_1, ν_2) degrees of freedom, are distributed (under the null hypothesis) following the pivotal quantities respectively:*

$$
\begin{aligned}
\overline{\Lambda}_s^{**} &= \frac{\left|W' M_{(s)}^{**} W\right|}{\left|W'\left(I - X\left(X'X\right)^{-1} X'\right) W\right|}, \\
&= \frac{\left|W'_{(-s)}\left(I - X_{(-s)}\left(X'_{(-s)} X_{(-s)}\right)^{-1} X'_{(-s)}\right) W_{(-s)}\right|}{\left|W'\left(I - X\left(X'X\right)^{-1} X'\right) W\right|} \\
\overline{\Lambda}_{\max}^{**} &= \sup_{s \in \mathcal{J}_{[1 \,:\, T]}} \left\{-T \ln\left(\overline{\Lambda}_s^{**}\right)\right\}, \\
\mathrm{LP}\left(\overline{\Lambda}^{**}\right) &= -2 \sum_{s \in \mathcal{J}_{[1 \,:\, T]}} \ln\left(pv\left[\overline{\Lambda}_s^{**}\right]\right), \\
pv\left[\overline{\Lambda}_s^{**}\right] &= G_F\left(\frac{1 - \overline{\Lambda}_s^{**}}{\overline{\Lambda}_s^{**}} \frac{T - K - n}{n} \mid n,\ T - K - n\right)
\end{aligned}
$$

where $M_{(s)}^{**} = I - X_{(d_s)}\left(X'_{(d_s)} X_{(d_s)}\right)^{-1} X'_{(d_s)}$, *$W$ is defined by* (2.2) *and $X_{(-s)}$ and $W_{(-s)}$ are obtained from X and W respectively, by deleting the sth row.*

Proof. Following the arguments of Theorems 1-2, we can write the statistic Λ_s^{**} under the null hypothesis and assumption (2.2) as follows:

$$
\begin{aligned}
\Lambda_s^{**} &= \frac{\left|U' M_{(s)}^{**} U\right|}{\left|U'\left(I - X\left(X'X\right)^{-1} X'\right) U\right|} = \frac{\left|W' M_{(s)}^{**} W\right|}{\left|W'\left(I - X\left(X'X\right)^{-1} X'\right) W\right|}, \\
&= \frac{\left|U'_{(-s)}\left(I - X_{(-s)}\left(X'_{(-s)} X_{(-s)}\right)^{-1} X'_{-s}\right) U_{-s}\right|}{\left|U'\left(I - X\left(X'X\right)^{-1} X'\right) U\right|} \\
&= \frac{\left|W'_{(-s)}\left(I - X_{(-s)}\left(X'_{(-s)} X_{(-s)}\right)^{-1} X'_{(-s)}\right) W_{(-s)}\right|}{\left|W'\left(I - X\left(X'X\right)^{-1} X'\right) W\right|}
\end{aligned}
$$

where $U_{(-s)}$ is obtained from U by deleting the sth row. ■

Under the null hypothesis and normal errors, the following holds exactly:

$$
\frac{1 - \Lambda_s^{**}}{\Lambda_s^{**}} \frac{T - K - n}{n} \sim \mathrm{F}\left(n,\ T - K - n\right).
$$

Λ_s^{**} and $\Lambda_{\max}^{**}$ were suggested by Wilks (1963) for the location-scale model; Wilks (1963) proposed the application of Boole-Bonferroni bounds p-values for $\Lambda_{\max}^{**}$. Bonferonni type procedures typically involve dividing the overall size of the (joint) test by the number of underlying individual tests (here the dimension of the subset $\mathcal{J}_{[1\,:\,T]}$); obviously, such a procedure can become utterly conservative. Our interpretation of the latter outlier tests in terms of UL hypotheses allows their extensions to possibly non-Gaussian models with regressors. Indeed, Theorem 3 shows that MC tests based on Λ_s^{**}, $\Lambda_{\max}^{**}$ and $\mathrm{LP}(\Lambda^{**})$ may easily be obtained, as described above.

3.4 Dynamic Extensions

So far, we have considered model (2.5) where the dynamic coefficient λ is known. In this section, we show how to account for an unknown λ. Consider the MC p-values associated with the statistics introduced in the previous sections, namely Λ^*, $\mathrm{LP}(\Lambda^*)$, $-T\ln(\Lambda_s^*)$, $-T\ln(\Lambda)$, $-T\ln(\Lambda_s)$, $\mathrm{LP}(\Lambda)$, $\Lambda_{\max}$, $\Lambda_{\max}^{**}$, $\mathrm{LP}(\Lambda^{**})$, which we denote

$$\hat{p}_N(.;\lambda), \tag{3.54}$$

where the conditioning on λ is now explicit, to set the framework for the unknown λ case.

We follow the consistent set maximized MC (CSMMC) test approach from Dufour and Kiviet (1996). First, obtain a consistent set estimate of level $(1-\alpha_1)$ for λ; for the predictive tests, this confidence set is taken over the stable sub-sample, while it is taken over the whole sample for the non predictive ones. These confidence sets will be denoted $\overline{CS}(\alpha_1;\lambda)$ and $\overline{CS}_1(\alpha_1;\lambda)$ where the subscript 1 refers to the estimate obtained over the stable sub-sample. The CSMMC methodology involves maximizing each of the p-value functions defined in (3.54) over the values of λ in these confidence sets. Let

$$\hat{p}_N^{\sup}(.) = \sup_{\lambda\in\overline{CS}(\alpha_1;\lambda)} \hat{p}_N(.;\lambda), \tag{3.55}$$

for the statistics Λ^*, $\mathrm{LP}(\Lambda^*)$, $-T\ln(\Lambda_s^*)$, $\Lambda_{\max}^{**}$, $\mathrm{LP}(\Lambda^{**})$, and

$$\hat{p}_N^{\sup}(.) = \sup_{\lambda\in\overline{CS}_1(\alpha_1;\lambda)} \hat{p}_N(.;\lambda), \tag{3.56}$$

for the statistics $-T\ln(\Lambda)$, $\mathrm{LP}(\Lambda)$, $-T\ln(\Lambda_s)$, $\Lambda_{\max}$; then the CSMMC tests critical regions of level α are respectively $\hat{p}_N^{\sup}(.) \le \alpha - \alpha_1$. An exact procedure to obtain a confidence set for λ is not readily available, so at this stage, we use the SURE-based Wald-type confidence set.

4 Empirical Applications

In this section we report the results of the above developed tests applied to the model of Arsenault et al. (1995), estimated with annual data sets for the Province of Québec (Canada), from 1962 to 2000, for six sectors of energy use: (i) residential, (ii) commercial, and (iii) four manufacturing sectors [paper and allied products, primary metals, refined petroleum and coal products, and all other manufacturing industries]. Energy sources considered are: electricity, natural gas, oil

Table 4. Total energy demand by sector

	Residential	Commercial	Paper and Allied	Primary Metal	Petroleum and Coal	Other
Constant	1.817	1.832	2.469	1.145	-0.501	2.984
	(5.49)	(4.45)	(3.30)	(2.66)	(-0.57)	(3.66)
Lagged dependent	0.614	0.400	0.562	0.664	0.970	0.525
	(7.47)	(4.14)	(7.26)	(8.88)	(18.94)	(5.17)
Energy Price	-0.285	-0.318	-0.047	-0.120	-0.123	-0.124
	(-4.85)	(-4.18)	(-1.72)	(-1.90)	(-1.40)	(-3.37)
Disposable income	0.191	-	-	-	-	-
	(1.81)					
Commercial GDP	-	0.546	-	-	-	-
		(4.46)				
Industrial GDP	-	-	0.364	0.401	0.184	0.289
			(4.26)	(4.36)	(1.36)	(4.50)
Heating degree-days	0.413	0.459	-	-	-	-
	(5.37)	(2.70)				
Observations	38	38	38	38	38	38

products and coal.[10] Aggregate energy quantities are obtained by adding the energy sources on the basis of their thermal content. Prices of energy sources are obtained by dividing total expenditures by total quantities.[11] Gaussian QMLE estimates are shown in tables 4 and 5; t-statistics are in parentheses. In the share equations system, natural gas is chosen as the numeraire.

In table 6, we report the results of the joint stability tests (from Dufour and Kiviet (1996)) applied to the demand equation; these tests are summarized in Appendix A. The 1962-1985 period which precedes oil and natural gas price deregulation in Québec was considered as the stable sample for the predictive test. In this table, $\widehat{\rho}_1$ gives (for reference purposes) the OLS estimate of ρ over the 1962-1985 subsample. The symbols a (or r) indicate that stability is accepted (or rejected) respectively, at level 5%. For manufacturing industries, the relevant cut-offs are $F(2.5\%; 15, 20) = 2.57$ and $F(7.5\%; 15, 20) = 1.99$. For residential and commercial sectors, the relevant cut-offs are $F(2.5\%; 15, 19) = 2.61$ and $F(7.5\%; 15, 19) = 2.01$.

[10] For the residential and commercial sectors, coal is not relevant so the share equations system reduces to a two-equation model.

[11] Data on expenditures and quantities by sector are from Statistics Canada. In particular, for the manufacturing industry, we use the annual Statistics Canada's publication on *consumption of purchased fuel and electricity in the manufacturing industries, catalogue 57-208* (which pertains to the annual census on the manufacturing sector). Publication of this catalogue was discontinued after 1984. Some variables were still available from statistics Canada (specifically, total energy expenditures for 1985, 1986 and 1990, and quantities of electricity for 1997-2000). Thus, we have completed the missing data [quantities and prices (1985-1990), quantities (1997-2000)] using other data sources. For more information, see *Nadhem Idoudi*, "La demande énergétique du secteur manufacturier québécois : analyse de la stabilité structurelle", Master Thesis, 2003, Université Laval.

Table 5. Estimated market share equations

Explanatory variables	Energy sources		
	Electricity	Oil	Coal
1. Paper and Allied			
Constant	0.056 (3.14)	-0.002 (-0.15)	0.009 (0.71)
Lagged dependent	0.936 (46.25)	0.936 (46.25)	0.936 (46.25)
Electricity price	-0.037 (-2.14)	0.045 (2.91)	-0.018 (-1.21)
Oil price	0.045 (2.91)	-0.109 (-5.04)	0.023 (1.64)
Coal price	-0.018 (-1.21)	0.023 (1.64)	0.013 (0.65)
2. Primary Metal			
Constant	0.037 (2.78)	-0.007 (-0.86)	0.005 (0.73)
Lagged dependent	0.953 (34.23)	0.953 (34.23)	0.953 (34.23)
Electricity price	-0.021 (-1.77)	0.020 (1.82)	-0.001 (-0.16)
Oil price	0.020 (1.82)	-0.041 (-2.10)	0.011 (1.90)
Coal price	-0.001 (-0.16)	0.011 (0.90)	-0.011 (-0.70)
3. Petroleum and Coal			
Constant	-0.031 (-0.64)	-0.003 (-0.41)	-
Lagged dependent	0.894 (17.88)	0.894 (17.88)	-
Electricity price	0.073 (1.58)	0.015 (2.27)	-
Oil price	0.015 (2.27)	-0.010 (-1.19)	-
Coal price	-	-	-
4. Other			
Constant	0.042 (3.23)	0.042 (3.09)	0.012 (2.08)
Lagged dependent	0.812 (32.00)	0.812 (32.00)	0.812 (32.00)
Electricity price	-0.017 (-1.98)	0.078 (7.12)	-0.008 (-1.80)
Oil price	0.078 (7.12)	-0.130 (-6.43)	0.0004 (0.07)
Coal price	-0.008 (-1.80)	0.0004 (0.07)	-0.005 (-1.04)
5. Residential			
Constant	0.077 (7.12)	-0.022 (-4.25)	-
Lagged dependent	0.934 (73.74)	0.934 (73.74)	-
Electricity price	-0.040 (-5.11)	0.044 (6.24)	-
Oil price	0.044 (6.24)	-0.049 (-6.05)	-
6. Commercial			
Constant	0.085 (4.29)	-0.010 (-0.95)	-
Lagged dependent	0.906 (43.61)	0.906 (43.61)	-
Electricity price	-0.033 (-2.54)	0.024 (2.45)	-
Oil price	0.024 (2.45)	-0.066 (-5.00)	-

Table 6. Stability joint F-tests in the demand equation

	Residential	Commercial	Paper & Allied	Primary Metal	Petroleum and Coal	Other
$\widehat{\rho}_1$	.440	.519	.440	.652	.973	.246
$\overline{CS}_1^*(2.5\%;\rho)$	$[.03,.99]$	$[.01,.99]$	$[.17,.77]$	$[.44,.99]$	$[.7,.99]$	$[.01,.99]$
$\overline{CS}_1^{**}(2.5\%;\rho)$	$[.01,.99]$	$[.01,.99]$	$[.19,.69]$	$[.51,.86]$	$[.74,.99]$	$[.03,.64]$
$\overline{CS}_1^a(2.5\%;\rho)$	$[.1,.77]$	$[.11,.89]$	$[.24,.63]$	$[.48,.82]$	$[.73,.99]$	$[.01,.51]$
$PC_{\min},\ \rho\in\overline{CS}_1^*$	.79	.10	.78	.94	.36	1.86
$PC_{\min},\ \rho\in\overline{CS}_1^{**}$	.79	.10	1.02	1.34	.36	2.85 r
$PC_{\min},\ \rho\in\overline{CS}_1^a$	.80	.10	1.27	1.48	.36	3.87 r
$PC_{\max},\ \rho\in\overline{CS}_1^*$	2.85	.90 a	2.45	1.90 a	.85 a	8.22
$PC_{\max},\ \rho\in\overline{CS}_1^{**}$	2.88	.90 a	2.45	1.90 a	.77 a	8.22
$PC_{\max},\ \rho\in\overline{CS}_1^a$	2.70	.64 a	2.45	1.90 a	.74 a	8.22

Note: $\widehat{\rho}_1$ is the point estimate of ρ in model (2.4) over 1962-1985; $\overline{CS}_1^*(2.5\%;\rho)$, $\overline{CS}_1^{**}(2.5\%;\rho)$, and $\overline{CS}_1^a(2.5\%;\rho)$ give the three consistent set estimators as specified in section A. The statistics $PC_{\min}$ and $PC_{\max}$ are defined in eq. (A.2). The symbol a (or r) indicates that stability is accepted (or rejected) respectively, at level 5%.

These results (for a level of 5%) show that stability is: (i) rejected for small and medium industries, (ii) not rejected for commercial, primary metal industries, refined petroleum and coal industries. The test is inconclusive for the residential sector and paper and allied industries.

The individual stability t-tests are reported in tables 7 and 8; the same confidence sets reported in table 6 are relevant for these tests. In these tables, $F_{\min}^s$ and $F_{\max}^s$ are the statistics defined in (A.4); $\overline{CS}_1^*(2.5\%;\rho)$, $\overline{CS}_1^{**}(2.5\%;\rho)$ and $\overline{CS}_1^a(2.5\%;\rho)$ are the three consistent set estimators of ρ as specified in Appendix A.

Tests results for the system of share equations are presented in table 9, and imply the following (in the context of a 5% significance level test). For the residential, commercial, primary metal industries, and the broad group "other manufacturing industries" sectors, all our tests fail to reject structural stability. In the case of the refined petroleum and coal products industries, the test LP (Λ^{**}) reveals significant outliers for the dates 1965, 1974 and 1979; not surprisingly, our tests are able to detect oil shock effects. For the paper and allied industries, the predictive break tests $-T\ln(\Lambda)$ and LP (Λ) are significant, with breaks detected in 1988, 1989, 1990. The outliers tests $\Lambda_{\max}^{**}$ and LP (Λ^{**}) confirm these findings for 1989. Special electricity price programs were guaranteed for this industry by the Québec government, from 1981-89; the break dates we find seem to detect adjustments due to price deregulation.

5 Conclusion

This paper considers finite sample motivated structural stability tests for the MLR model. Following Dufour and Kiviet (1996), our multivariate tests treat unknown dynamic coefficients

Table 7. Stability t-tests in the demand equation

		Residential		Commercial		Paper & Allied	
		$F^s_{\min}$	$F^s_{\max}$	$F^s_{\min}$	$F^s_{\max}$	$F^s_{\min}$	$F^s_{\max}$
1986	$\overline{CS}_1^*(2.5\%;\rho)$	.0004	4.20	.06	7.18	.32	1.48 a
	$\overline{CS}_1^{**}(2.5\%;\rho)$	.0004	4.32	.06	7.18	.36	1.48 a
	$\overline{CS}_1^a(2.5\%;\rho)$	.0004	3.72	.06	4.92	.50	1.48 a
1987	$\overline{CS}_1^*(2.5\%;\rho)$	.38	12.89	.002	1.34 a	.52	1.74 a
	$\overline{CS}_1^{**}(2.5\%;\rho)$	.38	12.89	.002	1.34 a	.74	1.74 a
	$\overline{CS}_1^a(2.5\%;\rho)$	2.16	12.81	.002	0.90 a	.96	1.74 a
1988	$\overline{CS}_1^*(2.5\%;\rho)$	.01	12.82	.002	1.90 a	2.46	7.39
	$\overline{CS}_1^{**}(2.5\%;\rho)$	.01	13.10	.002	1.90 a	3.46	7.39
	$\overline{CS}_1^a(2.5\%;\rho)$	.01	11.49	.002	1.12 a	4.36	7.39
1989	$\overline{CS}_1^*(2.5\%;\rho)$	.07	9.12	.0001	1.08 a	.0001	4.58
	$\overline{CS}_1^{**}(2.5\%;\rho)$	.07	9.54	.0001	1.08 a	.0001	4.49
	$\overline{CS}_1^a(2.5\%;\rho)$	.07	7.61	.0001	.59 a	.07	4.12
1990	$\overline{CS}_1^*(2.5\%;\rho)$	.006	6.45	.001	2.89 a	1.69	6.66
	$\overline{CS}_1^{**}(2.5\%;\rho)$	.006	6.55	.001	2.89 a	2.52	6.66
	$\overline{CS}_1^a(2.5\%;\rho)$	.03	6.00	.001	2.19 a	3.34	6.66
1991	$\overline{CS}_1^*(2.5\%;\rho)$	.07	4.84	.0004	.68 a	.38	6.60
	$\overline{CS}_1^{**}(2.5\%;\rho)$	.07	4.84	.0004	.68 a	.90	6.60
	$\overline{CS}_1^a(2.5\%;\rho)$	.62	4.79	.0004	.54 a	1.53	6.60
1992	$\overline{CS}_1^*(2.5\%;\rho)$	.0009	2.04 a	.0001	.79 a	.81	6.60
	$\overline{CS}_1^{**}(2.5\%;\rho)$	.0009	2.13 a	.0001	.79 a	1.48	6.60
	$\overline{CS}_1^a(2.5\%;\rho)$	.0009	1.71 a	.002	.59 a	2.22	6.60
1993	$\overline{CS}_1^*(2.5\%;\rho)$	.0009	2.95 a	.0004	1.63 a	.37	1.02 a
	$\overline{CS}_1^{**}(2.5\%;\rho)$	.0009	3.02 a	.0004	1.63 a	.41	1.02 a
	$\overline{CS}_1^a(2.5\%;\rho)$	.001	2.72 a	.003	1.18 a	.51	1.02 a
1994	$\overline{CS}_1^*(2.5\%;\rho)$	.0009	3.09 a	.01	1.87 a	.00	1.02 a
	$\overline{CS}_1^{**}(2.5\%;\rho)$	.0009	3.16 a	.01	1.87 a	.00	1.00
	$\overline{CS}_1^a(2.5\%;\rho)$	.0009	2.72 a	.01	1.25 a	.01	.92 a
1995	$\overline{CS}_1^*(2.5\%;\rho)$	.0001	9.18	.01	4.28	.0004	.53 a
	$\overline{CS}_1^{**}(2.5\%;\rho)$	.0001	9.30	.01	4.28	.0004	.50 a
	$\overline{CS}_1^a(2.5\%;\rho)$	.56	8.82	.01	3.16 a	.0004	.44 a
1996	$\overline{CS}_1^*(2.5\%;\rho)$	.04	4.12	.04	2.01 a	.003	.38 a
	$\overline{CS}_1^{**}(2.5\%;\rho)$	.04	4.32	.04	2.01 a	.003	.37 a
	$\overline{CS}_1^a(2.5\%;\rho)$	.04	3.42 a	.04	1.46 a	.003	.34 a
1997	$\overline{CS}_1^*(2.5\%;\rho)$	.002	6.05	.0004	0.96 a	.19	1.08 a
	$\overline{CS}_1^{**}(2.5\%;\rho)$	.002	6.10	.0004	0.96 a	.22	1.08 a
	$\overline{CS}_1^a(2.5\%;\rho)$	.40	5.76	.0004	0.51 a	.31	1.08 a
1998	$\overline{CS}_1^*(2.5\%;\rho)$	1.27	11.83	.09	5.10	.57	1.27 a
	$\overline{CS}_1^{**}(2.5\%;\rho)$	1.27	11.83	.09	5.10	.62	1.27 a
	$\overline{CS}_1^a(2.5\%;\rho)$	3.72	11.49	.09	3.68	.74	1.27 a
1999	$\overline{CS}_1^*(2.5\%;\rho)$	.0004	11.63	.01	2.13 a	.001	.33 a
	$\overline{CS}_1^{**}(2.5\%;\rho)$	.0004	11.63	.01	2.13 a	.01	.33 a
	$\overline{CS}_1^a(2.5\%;\rho)$	.0004	11.22	.01	1.61 a	.03	.32 a
2000	$\overline{CS}_1^*(2.5\%;\rho)$	.02	13.91	.004	2.07 a	.29	1.66 a
	$\overline{CS}_1^{**}(2.5\%;\rho)$	.02	14.31	.004	2.07 a	.49	1.66 a
	$\overline{CS}_1^a(2.5\%;\rho)$	.51	13.03	.004	1.63 a	.67	1.66 a

Table 8. Stability t-tests in the demand equation (continued)

		Primary Metal		Petroleum & Coal		Other	
		$F^s_{\min}$	$F^s_{\max}$	$F^s_{\min}$	$F^s_{\max}$	$F^s_{\min}$	$F^s_{\max}$
1986	$\overline{CS}^*_1(2.5\%;\rho)$	.0004	.77 a	.02	2.46 a	2.59	12.96
	$\overline{CS}^{**}_1(2.5\%;\rho)$	.0004	.58 a	.02	1.76 a	6.35 r	12.96
	$\overline{CS}^a_1(2.5\%;\rho)$	.0004	.49 a	.02	1.93 a	8.70 r	12.96
1987	$\overline{CS}^*_1(2.5\%;\rho)$	.003	.67 a	1.74	8.88	2.07	59.44
	$\overline{CS}^{**}_1(2.5\%;\rho)$	.003	.51 a	1.74	8.06	13.62 r	59.44
	$\overline{CS}^a_1(2.5\%;\rho)$	.003	.59 a	1.74	8.29	23.71 r	59.44
1988	$\overline{CS}^*_1(2.5\%;\rho)$	3.10	7.84	.004	1.93 a	.32	45.97
	$\overline{CS}^{**}_1(2.5\%;\rho)$	4.97	7.84	.004	1.36 a	.42	44.89
	$\overline{CS}^a_1(2.5\%;\rho)$	5.66	7.84	.004	1.51 a	3.80	45.97
1989	$\overline{CS}^*_1(2.5\%;\rho)$	2.02	6.70	.24	3.92	.07	16.40
	$\overline{CS}^{**}_1(2.5\%;\rho)$	3.57	6.70	.24	3.38	.07	15.68
	$\overline{CS}^a_1(2.5\%;\rho)$	4.20	6.70	.24	3.53	.07	16.40
1990	$\overline{CS}^*_1(2.5\%;\rho)$	.77	1.51 a	.46	3.84	1.76	33.40
	$\overline{CS}^{**}_1(2.5\%;\rho)$	1.12	1.51 a	.46	3.38 a	8.94 r	33.40
	$\overline{CS}^a_1(2.5\%;\rho)$	1.06	1.51 a	.46	3.49 a	14.82 r	33.40
1991	$\overline{CS}^*_1(2.5\%;\rho)$	1.21	3.31 a	.001	.84 a	.25	12.04
	$\overline{CS}^{**}_1(2.5\%;\rho)$	1.99	3.31 a	.001	.84 a	2.34	12.04
	$\overline{CS}^a_1(2.5\%;\rho)$	2.28	3.31 a	.001	.12 a	4.32	12.04
1992	$\overline{CS}^*_1(2.5\%;\rho)$	1.49	3.16 a	.16	3.06 a	.006	9.54
	$\overline{CS}^{**}_1(2.5\%;\rho)$	2.25	3.16 a	.16	2.62 a	.38	9.36
	$\overline{CS}^a_1(2.5\%;\rho)$	2.43	3.16 a	.16	2.75 a	1.39	9.54
1993	$\overline{CS}^*_1(2.5\%;\rho)$	.48	1.61 a	.20	3.42 a	.0001	15.60
	$\overline{CS}^{**}_1(2.5\%;\rho)$	.85	1.61 a	.20	2.95 a	.47	15.37
	$\overline{CS}^a_1(2.5\%;\rho)$	1.00	1.61 a	.20	3.06 a	2.01	15.60
1994	$\overline{CS}^*_1(2.5\%;\rho)$	.12	.76 a	.01	1.44 a	.02	12.11
	$\overline{CS}^{**}_1(2.5\%;\rho)$	.23	.76 a	.01	1.14 a	.14	11.63
	$\overline{CS}^a_1(2.5\%;\rho)$	.17	.75 a	.01	1.21 a	.00	12.11
1995	$\overline{CS}^*_1(2.5\%;\rho)$	.00	.59 a	.04	1.84 a	.00	7.07
	$\overline{CS}^{**}_1(2.5\%;\rho)$	.02	.53 a	.04	1.56 a	.31	6.86
	$\overline{CS}^a_1(2.5\%;\rho)$	.01	.49 a	.04	1.63 a	.01	7.07
1996	$\overline{CS}^*_1(2.5\%;\rho)$	.13	.34 a	.98	4.32	1.08	9.18
	$\overline{CS}^{**}_1(2.5\%;\rho)$	.19	.34 a	.98	3.96	2.40	9.12
	$\overline{CS}^a_1(2.5\%;\rho)$	.16	.30 a	.98	4.04	.07	9.18
1997	$\overline{CS}^*_1(2.5\%;\rho)$	.37	.88 a	.03	2.22 a	.07	4.97
	$\overline{CS}^{**}_1(2.5\%;\rho)$	.53	.88 a	.03	1.84 a	.07	4.66
	$\overline{CS}^a_1(2.5\%;\rho)$	.46	.88 a	.03	1.93 a	.31	4.97
1998	$\overline{CS}^*_1(2.5\%;\rho)$	.39	.77 a	.0004	1.32 a	4.45	18.49
	$\overline{CS}^{**}_1(2.5\%;\rho)$	.53	.77 a	.0004	1.08 a	7.61 r	18.49
	$\overline{CS}^a_1(2.5\%;\rho)$	.46	.77 a	.0004	1.14 a	.01	18.49
1999	$\overline{CS}^*_1(2.5\%;\rho)$	1.53	2.75 a	.0006	1.16 a	1.58	29.38
	$\overline{CS}^{**}_1(2.5\%;\rho)$	1.98	2.75 a	.0006	.88 a	4.97	29.05
	$\overline{CS}^a_1(2.5\%;\rho)$	1.79	2.75 a	.0006	.94 a	.05	29.38
2000	$\overline{CS}^*_1(2.5\%;\rho)$	2.79	5.61	.0004	.98 a	.05	7.56
	$\overline{CS}^{**}_1(2.5\%;\rho)$	4.53	5.61	.0004	.77 a	.05	7.07
	$\overline{CS}^a_1(2.5\%;\rho)$	4.16	5.61	.0004	.82 a	.05	7.567.56

Table 9. Structural change tests in the share equations system

	Residential	Commercial	Paper and Allied
$\overline{CS}(2.5\%;\lambda)$	[.907;.961]	[.861;.951]	[.891;.981]
$\hat{p}_N^{\text{sup}}(\Lambda^*)$	.864	.210	.050
$\hat{p}_N^{\text{sup}}(\text{LP}(\Lambda^*))$	.836	.106	.260
$\overline{CS}_1(2.5\%;\lambda)$	[.685;.959]	[.6;.999]†	[.841;.999]
$\hat{p}_N^{\text{sup}}(-T\ln(\Lambda))$	.552	.844	**.016**
Dates Λ_s-significant at 5%	∅	∅	1988, 1989, 1990
$\hat{p}_N^{\text{sup}}(\Lambda_{\max})$	.300	.596	.054
$\hat{p}_N^{\text{sup}}(\text{LP}(\Lambda))$	.506	.806	**.012**
$\hat{p}_N^{\text{sup}}(\Lambda_{\max}^{**})$	.758	.502	**.004**
$\hat{p}_N^{\text{sup}}(\text{LP}(\Lambda^{**}))$	.934	.098	**.002**
Dates Λ_s^{**}-significant at 5%	∅	∅	1966, 1989
	Primary Metal	Petroleum & Coal	Other
$\overline{CS}(2.5\%;\lambda)$	[.874;.995]	[.782;1]	[.756;.868]
$\hat{p}_N^{\text{sup}}(\Lambda^*)$	.906	.152	.404
$\hat{p}_N^{\text{sup}}(\text{LP}(\Lambda^*))$	.814	.414	.682
$\overline{CS}_1(2.5\%;\lambda)$	[.638;.996]	[.645;1]	[.691;.887]
$\hat{p}_N^{\text{sup}}(-T\ln(\Lambda))$	.782	1.00	.490
Dates Λ_s-significant at 5%	∅	∅	∅
$\hat{p}_N^{\text{sup}}(\Lambda_{\max})$	.228	.996	.528
$\hat{p}_N^{\text{sup}}(\text{LP}(\Lambda))$	.428	1	.438
$\hat{p}_N^{\text{sup}}(\Lambda_{\max}^{**})$	.848	.072	.186
$\hat{p}_N^{\text{sup}}(\text{LP}(\Lambda^{**}))$	.594	**.016**	.044
Dates Λ_s^{**}-significant at 5%	∅	1965, 1974, 1979	∅

Note: Λ^* and LP(Λ^*) are defined in Theorem 1 (our variants of Bai et al. (1998)'s tests); the associated consistent confidence set estimate $\overline{CS}(2.5\%;\lambda)$ defined in section 3.1 is based on the entire sample. Λ, $\Lambda_{\max}$ and LP(Λ) and Λ_s are the predictive test statistics defined in Theorem 2; the associated consistent confidence set estimate $\overline{CS}_1(2.5\%;\lambda)$ is based on the stable sample 1962-85. "Dates Λ_s-significant at 5%" are the dates s such that $\hat{p}_N^{\text{sup}}(-T\ln(\Lambda_s))$ is less than 2.5%; see sections 3.2 and 3.4. $\Lambda_{\max}^{**}$, LP(Λ^{**}) and Λ_s^{**} are defined in Theorem 3 (our variants of Wilks (1963)' test statistics).

Table 10. CSMMC tests for individual dates in the share equations system : the maximum p-value

	Residential	Commercial	Paper & Allied	Primary Metal	Petroleum and Coal	Other
1986	.984	.136	.658	.460	1.00	.178
1987	.226	.242	.706	.144	1.00	.832
1988	.070	.856	**.014**	.766	.960	.764
1989	.038	.948	**.016**	.776	.954	.236
1990	.750	.976	**.008**	.388	.956	1.00
1991	.346	.952	.042	.028	.994	.806
1992	1.00	.406	.598	.506	.852	.430
1993	.958	.854	.886	.438	.962	.538
1994	.772	.884	.942	1.00	.956	.598
1995	.788	.868	.868	.246	.946	.778
1996	.904	.892	.970	.984	1.00	.530
1997	.818	.914	.910	.422	1.00	.430
1998	.296	.986	.300	.604	1.00	.432
1999	.992	.900	.684	.972	.326	.860
2000	.998	.396	.382	.582	.996	.088

Note: Numbers reported are the maximized p-values (in %), as defined in (3.55), associated with Λ_s, the statistic defined in Theorem 2 with the consistent confidence set estimate $\overline{CS}_1(2.5\%;\lambda)$ based on the stable subsample 1962-85. Stability is rejected for a given year at the 5% level if the corresponding maximum p-value is less than 2.5%.

as a nuisance parameter, which we account for through a two-stage consistent set maximized Monte Carlo (CSMMC) procedure [Dufour (2002)]. We propose three types of structural change tests: exact versions of Bai et al. (1998)'s parameter constancy test, alternative multivariate predictive test approaches, and extensions of Wilks (1963)'s multivariate outliers test. These tests are applied to the energy demand model analyzed by Arsenault et al. (1995). Commonly used two-level energy demand models consist of single equations which estimate total demand and a system of share equations (by energy source). Both specifications are dynamic and data sets are annual; sample size constraints are thus severe, which underscore the need for finite sample inference. The procedures we propose have potential useful applications in statistics, econometrics and finance.

A Appendix: Univariate Structural Change Tests

In this section, we describe the structural change tests of Dufour and Kiviet (1996), as they apply to the demand equations specified in (2.4). The null model is (2.4); the alternative hypothesis allows for break in all parameters, ρ, a_1, ..., a_4 and even σ^2 after a specific date, say T_1. For a given value of ρ, say ρ_0, a predictive F-statistic may be obtained as follows

$$PC\left(\rho_0\right) = \frac{T_1 - k}{T_2}\left\{\frac{S_0\left(\rho_0\right) - S_1\left(\rho_0\right)}{S_1\left(\rho_0\right)}\right\},$$

where $T_2 = T - T_1$, $S_0\left(\rho_0\right)$ is the OLS-based residual sum of squares [RSS] associated with

$$\ln\left(\mathrm{TE}_t\right) - \rho_0 \ln\left(\mathrm{TE}_{t-1}\right) = x_t'\left(\rho_0\right)\beta + u_t,\ t = 1, ..., T,$$

and $S_1\left(\rho_0\right)$ is the RSS associated with the latter model estimated over the stable subsample

$$\ln\left(\mathrm{TE}_t\right) - \rho_0 \ln\left(\mathrm{TE}_{t-1}\right) = x_t'\left(\rho_0\right)\beta_1 + u_t,\ t = 1, ..., T_1. \tag{A.1}$$

$PC\left(\rho_0\right) \sim F(T_2, T_1 - k)$ since ρ_0 is known. To account for an unknown ρ, Dufour and Kiviet's α-level bound test proceeds as follows. Reject stability if $PC_{\min} \geq F(\alpha - \alpha_1; T_2, T_1 - k)$; accept stability if $PC_{\max} < F(\alpha + \alpha_1; T_2, T_1 - k)$; consider the test inconclusive otherwise, where

$$PC_{\min} = \inf_{\rho_0 \in \overline{CS}_1(\alpha_1;\rho)} PC\left(\rho_0\right), \quad PC_{\max} = \sup_{\rho_0 \in \overline{CS}_1(\alpha_1;\rho)} PC\left(\rho_0\right), \tag{A.2}$$

and $\overline{CS}_1(\alpha_1; \rho)$ is a confidence set for ρ over the stable subsample with level $(1 - \alpha_1)$, $0 < \alpha_1 < \alpha < 1$, *i.e.* $\Pr[\rho \in \overline{CS}_1(\alpha_1; \rho)] \geq 1 - \alpha_1$. Dufour and Kiviet show that this test is equivalent to assessing the joint significance of the dummy variables in the augmented regression

$$\ln\left(\mathrm{TE}_t\right) - \rho_0 \ln\left(\mathrm{TE}_{t-1}\right) = x_t'\left(\rho_0\right)\beta + \sum_{s=T_1+1}^{T} d_{ts}\gamma_s + u_t,\ t = 1, ..., T, \tag{A.3}$$

where the time-dummy variables d_{ts} are defined in (3.29). To identify break points, let

$$t_s(\rho_0) = \frac{\ln\left(\mathrm{TE}_s\right) - \rho_0 \ln\left(\mathrm{TE}_{s-1}\right) - x_s'\left(\rho_0\right)\widehat{\beta}_1}{\frac{S_1(\rho_0)}{T_1-k}\left[1 + x_s'\left(\rho_0\right)\left(z_{(1)}'z_{(1)}\right)^{-1} x_s\left(\rho_0\right)\right]^{1/2}},\ s = T_1 + 1, ..., T\ ,$$

where $\widehat{\beta}_1$ and $z_{(1)}$ denote respectively the OLS estimator and associated matrix of regressors from (the stable) regression (A.1). If ρ_0 is known, then $(t_s(\rho_0))^2 \sim F(1, T_1 - k)$. To account for an unknown ρ, Dufour and Kiviet's sequence of bound tests yields the following. Reject stability when $F_{\min}^s \geq F(\alpha - \alpha_1; 1, T_1 - k)$; accept stability when $F_{\max}^s < F(\alpha + \alpha_1; 1, T_1 - k)$, and consider the test inconclusive otherwise, where

$$F_{\min}^s = \inf_{\rho_0 \in \overline{CS}_1(\alpha_1;\rho)} (t_s(\rho_0))^2, \quad F_{\max}^s = \sup_{\rho_0 \in \overline{CS}_1(\alpha_1;\rho)} (t_s(\rho_0))^2. \tag{A.4}$$

To obtain $\overline{CS}_1(\alpha_1; \rho)$, we consider the simple OLS-based confidence interval (which is not exact), and the two exact confidence sets which invert the pivotal LR-type statistics from Dufour and Kiviet (1996) (their equations 3.7, and 3.8). The asymptotic confidence set is denoted $\overline{CS}_1^a(\alpha_1; \rho)$, and the latter two exact ones are denoted $\overline{CS}_1^*(\alpha_1; \rho)$ and $\overline{CS}_1^{**}(\alpha_1; \rho)$ respectively.

B Appendix: An alternative expression for Bai et al. (1998)'s Wald statistic

Bai et al. (1998) write the model (3.7) in the form:

$$Y_t = \left(X_t' \otimes I_n\right) vec\left(B'\right) + D_{ts}\left(X_t' \otimes I_n\right)\left(Q_X \otimes I_n\right) vec\left(\Delta_s'\right) + U_t.$$

Let $\beta_s = \left(\left(vec\left(B'\right)\right)', \left(vec\left(\Delta_s'\right)\right)'\right)'$; in this context, the null hypothesis H_{0s}^* (2) corresponds to

$$\Delta_s = 0 \Leftrightarrow R^*\Theta_s = 0 \Leftrightarrow \Psi\beta_s = 0 \Leftrightarrow \left(I_n \otimes R^*\right)\theta_s = 0 \tag{B.5}$$

where $R^* = \left[\begin{array}{cc} \mathbf{0}_{q_X \times K} & I_{q_X} \end{array}\right]$, $\Psi = \left[\begin{array}{cc} \mathbf{0}_{(nq_X \times Kn)} & I_{nq_X} \end{array}\right]$ is a selection matrix of zeros and ones such that $\Psi\beta_s = vec\left(\Delta_s'\right)$, and $\theta_s = vec\left(\Theta_s\right)$. Let $\hat{\Theta}_s = \left[\hat{B}_s', \ \hat{\Delta}_s'\right]'$ denote the OLS estimate of Θ_s in (3.7). Then $\hat{\beta}_s = \left(\left(vec\left(\hat{B}_s'\right)\right)', \left(vec\left(\hat{\Delta}_s'\right)\right)'\right)'$ and $\hat{\theta}_s = vec(\hat{\Theta}_s)$ are the OLS estimate of β_s and θ_s, and the residual $\hat{U}_s^*$ from Theorem 1 obtains as:

$$\hat{U}_s^* = Y - X\hat{B}_s - D_s\hat{\Delta}_s. \tag{B.6}$$

With this notation, Bai et al. (1998)'s Wald statistic is

$$\begin{aligned}
\mathcal{W}_s^* &= T\left(\Psi\hat{\beta}_s\right)'\left[\Psi\left(\sum_{t=1}^{T}\mathfrak{Z}_{st}\left(\mathcal{S}_s^*\right)^{-1}\mathfrak{Z}_{st}'\right)^{-1}\Psi'\right]^{-1}\left(\Psi\hat{\beta}_s\right) \\
&= Tvec\left(\hat{\Delta}_s'\right)'\left[\Psi\left(\sum_{t=1}^{T}\mathfrak{Z}_{st}\left(\mathcal{S}_s^*\right)^{-1}\mathfrak{Z}_{st}'\right)^{-1}\Psi'\right]^{-1} vec\left(\hat{\Delta}_s'\right) \\
&= T\left[\left(I_n \otimes R^*\right)\hat{\theta}_s\right]'\left[\left(I_n \otimes R^*\right)\left(\mathcal{S}_s^* \otimes \left(Z_s'Z_s\right)^{-1}\right)\left(I_n \otimes R^*\right)'\right]^{-1}\left[\left(I_n \otimes R^*\right)\hat{\theta}_s\right] \\
&= T\left[\left(I_n \otimes R^*\right)\hat{\theta}_s\right]'\left[\left(\mathcal{S}_s^* \otimes R^*\left(Z_s'Z_s\right)^{-1}R^{*\prime}\right)\right]^{-1}\left[\left(I_n \otimes R^*\right)\hat{\theta}_s\right]
\end{aligned}$$

where $\mathcal{S}_s^* = \hat{U}_s^{*\prime}\hat{U}_s^*$ is as defined in Theorem 1 and $\mathfrak{Z}_{st}' = \left[\begin{array}{cc} \left(X_t' \otimes I_n\right) & D_{ts}\left(X_t' \otimes I_n\right)Q_X \otimes I_n \end{array}\right]$. Observe that $Z_s'Z_s = \left[\begin{array}{cc} X'X & X'D_s \\ D_s'X & D_s'D_s \end{array}\right]$. Applying the formula for the inverse of partitioned matrices, we see that

$$R^*\left(Z_s'Z_s\right)^{-1}R^{*\prime} = \left(D_s'\left(I_T - P_X\right)D_s\right)^{-1},$$

which implies that[12]

$$\begin{aligned}
\mathcal{W}_s^* &= T\left[\left(I_n \otimes R^*\right)\hat{\theta}_s\right]'\left[\left(\left(\mathcal{S}_s^*\right)^{-1} \otimes \left(D_s'\left(I_T - X\left(X'X\right)^{-1}X'\right)D_s\right)\right)\right]\left[\left(I_n \otimes R^*\right)\hat{\theta}_s\right] \\
&= T\left(vec\hat{\Delta}_s\right)'\left[\left(\mathcal{S}_s^*\right)^{-1} \otimes \left(D_s'\left(I_T - X\left(X'X\right)^{-1}X'\right)D_s\right)\right]\left(vec\hat{\Delta}_s\right) \\
&= T\,\mathrm{trace}\left[\left(\mathcal{S}_s^*\right)^{-1}\hat{\Delta}_s'\left(D_s'\left(I_T - X\left(X'X\right)^{-1}X'\right)D_s\right)\hat{\Delta}_s\right].
\end{aligned}$$

[12] We use the following result from matrix algebra: $\mathsf{trace}\left(A_1A_2A_3A_4\right) = \left(vecA_4\right)'\left(A_1 \otimes A_3'\right)\left(vecA_2'\right)$, where $A_1A_2A_3A_4$ is the product of the matrices A_1, A_2, A_3 and A_4.

From (B.6) $D_s\hat{\Delta}_s = Y - X\hat{B}_s - \hat{U}_s^*$, so

$$\left(I_T - X\left(X'X\right)^{-1}X'\right)D_s\hat{\Delta}_s = \left(I_T - X\left(X'X\right)^{-1}X'\right)\left(Y - X\hat{B}_s - \hat{U}_s^*\right).$$

It follows that

$$\begin{aligned}
\hat{\Delta}_s'\left(D_s'\left(I_T - X\left(X'X\right)^{-1}X'\right)D_s\right)\hat{\Delta}_s &= Y'\left(I_T - X\left(X'X\right)^{-1}X'\right)Y \\
&\quad -\left(X\hat{B}_s + \hat{U}_s^*\right)'\left(I_T - X\left(X'X\right)^{-1}X'\right)Y \\
&\quad -Y'\left(I_T - X\left(X'X\right)^{-1}X'\right)\left(X\hat{B}_s + \hat{U}_s^*\right) \\
&\quad +\left(X\hat{B}_s + \hat{U}_s^*\right)'\left(I_T - X\left(X'X\right)^{-1}X'\right)\left(X\hat{B}_s + \hat{U}_s^*\right) \\
&= Y'\left(I_T - X\left(X'X\right)^{-1}X'\right)Y \\
&\quad -\hat{U}_s^{*\prime}\left(I_T - X\left(X'X\right)^{-1}X'\right)Y \\
&\quad -Y'\left(I_T - X\left(X'X\right)^{-1}X'\right)\hat{U}_s^* + \hat{U}_s^{*\prime}\hat{U}_s^* \\
&= \mathcal{S}_0 - \hat{U}_s^{*\prime}\hat{U}^0 - \hat{U}^{0\prime}\hat{U}_s^* + \hat{U}_s^{*\prime}\hat{U}_s^*
\end{aligned}$$

where $\hat{U}^0$ and $\mathcal{S}_0$ are the residual and sum of squared residuals associated with (2.1) as defined in Theorem 1. Finally, on observing that $\hat{U}_s^{*\prime}\hat{U}_0 = \hat{U}_s^{*\prime}\left(\hat{U}_s^* + \hat{U}_0 - \hat{U}_s^*\right) = \hat{U}_s^{*\prime}\hat{U}_s^* + \hat{U}_s^{*\prime}\left(\hat{U}_0 - \hat{U}_s^*\right)$, and $\hat{U}_s^{*\prime}\left(\hat{U}_0 - \hat{U}_s^*\right) = 0$ since $\hat{U}_s^*$ is orthogonal to Z_s and

$$\hat{U}_0 - \hat{U}_s^* = Z_s(Z_s'Z_s)^{-1}R^{*\prime}[R^*(Z_s'Z_s)^{-1}R^{*\prime}]^{-1}R^*(Z_s'Z_s)^{-1}Z_s',$$

we obtain the desired result $\mathcal{W}_s^* = T\,\text{trace}\left[(\mathcal{S}_s^*)^{-1}(\mathcal{S}_0 - \mathcal{S}_s^*)\right]$. An alternative proof may also be obtained from Berndt and Savin (1977) and Gouriéroux et al. (1995) who derive an expression for Wald statistics associated with a UL hypothesis of which H_{0s}^* is a special case. For completeness, we note that the hypothesis H_{0s}^* under consideration is a special case of the one analyzed in Bai et al. (1998), which include SURE-type restrictions on the coefficients matrix Δ_s.

References

Anderson, T. W. (1984), *An Introduction to Multivariate Statistical Analysis*, second edn, John Wiley & Sons, New York.

Andrews, D. W. K. (1993), 'Tests for parameter instability and structural change with unknown change point', *Econometrica* **61**, 821–856.

Andrews, D. W. K. (2003), 'Tests for parameter instability and structural change with unknown change point: A corrigendum', *Econometrica* **71**, 395–397.

Andrews, D. W. K. and Ploberger, W. (1994), 'Optimal tests when a nuisance parameter is present only under the alternative', *Econometrica* **62**, 1383–1414.

Arsenault, E., Bernard, J.-T., Carr, C. W. and Genest-Laplante, E. (1995), 'A total energy demand model of Québec: Forecasting properties', *Energy Economics* **17(2)**, 163–171.

Bai, J., Lumsdaine, R. L. and Stock, J. H. (1998), 'Testing and dating common breaks in multivariate time series', *The Review of Economic Studies* **65(3)**, 395–432.

Bai, J. and Perron, P. (1998), 'Estimating and testing linear models with multiple structural changes', *Econometrica* **66**, 71–78.

Bai, J. and Perron, P. (2003), 'Computation and analysis of multiple structural change models', *Journal of Applied Econometrics* **18**, 1–22.

Banerjee, A., Lazarova, S. and Urga, G. (1998), Bootstrapping sequential tests for multiple structural breaks, Technical report, Center for Economics and Forecasting, London Business School, London, UK.

Barber, B. M. and Lyon, J. D. (1997), 'Detecting long-run abnormal stock returns: The empirical power and specification of test statistics', *Journal of Financial Economics* **43**, 341–372.

Barnard, G. A. (1963), 'Comment on 'The spectral analysis of point processes' by M. S. Bartlett', *Journal of the Royal Statistical Society, Series B* **25**, 294.

Bekaert, G., Harvey, C. R. and Lumsdaine, R. L. (2002), 'Dating the integration of world equity markets', *Journal of Financial Economics* **65**, 203–248.

Berndt, E. R. (1991), *The Practice of Econometrics: Classic and Contemporary*, Addison-Wesley, Reading (MA).

Berndt, E. R. and Savin, N. E. (1977), 'Conflict among criteria for testing hypotheses in the multivariate linear regression model', *Econometrica* **45**, 1263–1277.

Binder, J. J. (1985*a*), 'Measuring the effects of regulation with stock price data', *Rand Journal of Economics* **16**, 167–183.

Binder, J. J. (1985*b*), 'On the use of the multivariate regression model in event studies', *Journal of Accounting Research* **23(1)**, 370–383.

Bohi, D. R. (1981), *Analyzing Demand Behavior: A Study of Energy Elasticities*, John Hopkins Press, Baltimore, MD.

Bohi, D. R. and Zimmerman, M. B. (1984), 'An update on econometric studies of energy demand behavior', *Annual Review of Energy* **9**, 105–154.

Brown, S. J. and Warner, J. B. (1980), 'Measuring security-price performance', *Journal of Financial Economics* **8**, 205–258.

Brown, S. J. and Warner, J. B. (1985), 'Using daily stock returns: The case of event studies', *Journal of Financial Economics* **14**, 3–31.

Cantrell, R. S. J., Burrows, P. M. and Vuong, Q. H. (1991), 'Interpretation and use of generalized Chow tests', *International Economic Review* **32**, 725–741.

Caroni, C. and Prescott, P. (1992), 'Sequential application of Wilks multivariate outlier test', *Applied Statistics* **41**, 355–364.

Christiano, L. J. (1992), 'Searching for a break in GNP', *Journal of Business and Economic Statistics* **10**, 237–249.

Dhrymes, P. J., Howrey, P., Hymans, S. H., Kmenta, J., Leamer, E. E., Quandt, R. E., Ramsey, J. B., Shapiro, H. T. and Zarnowitz, V. (1972), 'Criteria for evaluation of econometric models', *Annals of Economic and Social Measurement* **1(3)**, 291–324.

Diebold, F. X. and Chen, C. (1996), 'Testing structural stability with endogenous break-point: A size comparison of analytic and bootstrap procedures', *Journal of Econometrics* **70**, 221–241.

Dufour, J.-M. (1980), 'Dummy variables and predictive tests for structural change', *Economics Letters* **6**, 241–247.

Dufour, J.-M. (2002), 'Monte Carlo tests with nuisance parameters: A general approach to finite-sample inference and nonstandard asymptotics in econometrics', *Journal of Econometrics* **forthcoming**.

Dufour, J.-M. and Khalaf, L. (2001), Monte Carlo test methods in econometrics, *in* B. Baltagi, ed., 'Companion to Theoretical Econometrics', Blackwell Companions to Contemporary Economics, Basil Blackwell, Oxford, U.K., chapter 23, pp. 494–519.

Dufour, J.-M. and Khalaf, L. (2002*a*), 'Exact tests for contemporaneous correlation of disturbances in seemingly unrelated regressions', *Journal of Econometrics* **106**(1), 143–170.

Dufour, J.-M. and Khalaf, L. (2002*b*), 'Simulation based finite and large sample tests in multivariate regressions', *Journal of Econometrics* **111**(2), 303–322.

Dufour, J.-M., Khalaf, L. and Beaulieu, M.-C. (2003), 'Exact skewness-kurtosis tests for multivariate normality and goodness-of-fit in multivariate regressions with application to asset pricing models', *Oxford Bulletin of Economics and Statistics* **65**, 891–906.

Dufour, J.-M., Khalaf, L., Bernard, J.-T. and Genest, I. (2004), 'Simulation-based finite-sample tests for heteroskedasticity and ARCH effects', *Journal of Econometrics* **122**, 317–347.

Dufour, J.-M. and Kiviet, J. F. (1996), 'Exact tests for structural change in first-order dynamic models', *Journal of Econometrics* **70**, 39–68.

Dufour, J.-M. and Kiviet, J. F. (1998), 'Exact inference methods for first-order autoregressive distributed lag models', *Econometrica* **66**, 79–104.

Dunstan, R. H. and Schmidt, R, H. (1988), 'Structural changes in residential energy demand', *Energy Economics* **10**, 206–212.

Dwass, M. (1957), 'Modified randomization tests for nonparametric hypotheses', *Annals of Mathematical Statistics* **28**, 181–187.

Fisher, R. A. (1932), *Statistical Methods for Research Workers*, Oliver and Boyd, Edinburgh.

Gouriéroux, C., Monfort, A. and Renault, E. (1995), Inference in factor models, *in* G. S. Maddala, P. C. B. Phillips and T. N. Srinivasan, eds, 'Advances in Econometrics and Quantitative Economics', Blackwell, Oxford, U.K., chapter 13, pp. 311–353.

Hadi, A. S. (1992), 'Identifiying multiple outliers in multivariate data', *Journal of the Royal Statistical Society Series B* **54(3)**, 761–771.

Hawdon, D. (1992), *Energy Demand: Evidence and Expectations*, Surrey University Press, London.

Hooper, J. W. and Zellner, A. (1961), 'The error of forecast for multivariate regression models', *Econometrica* **29(1)**, 544–555.

Hughes, J. S. and Ricks, W. E. (1984), 'Accounting for detail land sales analysis of a mandated change', *Journal of Accounting and Economics* **6**, 101–132.

Jayatissa, W. A. (1976), 'Criteria for evaluation of econometric models: A correction', *Annals of Economic and Social Measurement* **5**(1), 161.

Jayatissa, W. A. and Farebrother, R. W. (1977), 'A predictive test for the reduced form model', *Annals of Economic and Social Measurement* **6**(4), 473–476.

Kariya, T. (1985), *Testing in the Multivariate General Linear Model*, number 22 *in* 'Economic Research Series, The Institute of Economic Research, Hitotsubashi University, Japan', Kinokuniya Company Ltd., Tokyo.

Madlener, R. (1996), 'Econometric analysis of residential energy demand: A survey', *Journal of Energy Literature* **II(2)**, 3–32.

McAvinchey, I. D. and Yannopoulos, A. (2003), 'Stationarity, structural change and specification in a demand system: The case of energy', *Energy Economics* **25**, 65–92.

Pearson, K. (1933), 'On a method of determining whether a sample of size n supposed to have been drawn from a parent population having a known probability integral has probably been drawn at random', *Biometrika* **25**, 379–410.

Pindyck, R. S. (1999), 'The long run evolution of energy prices', *The Energy Journal* **20**, 1–27.

Qiu, P. and Hawkins, D. (2001), 'A rank-based multivariate CUSUM procedure', *Technometrics* **43**, 120–132.

Qiu, P. and Hawkins, D. (2003), 'A nonparametric multivariate CUSUM procedure for detecting shifts in all directions', *The Statistician* **52(2)**, 151–164.

Rao, C. R. (1973), *Linear Statistical Inference and its Applications*, second edn, John Wiley & Sons, New York.

Salinger, M. (1992), 'Standard errors in event studies', *Journal of Financial and Quantitative Analysis* **27**, 39–53.

Schipper, K. and Thompson, R. (1985), 'The impact of merger-related regulations using exact distributions of test statistics', *Journal of Accounting Research* **23**, 408–415.

Slade, M. E., Kolstad, C. D. and Weiner, R. J. (1993), Buying energy and nonfuel minerals: Final, derived and speculative demand, *in* A. V. Kneese and J. L. Sweeney, eds, 'Handbook of Natural Resources and Energy Economics, Volume III', Elsevier, Amsterdam: North-Holland, chapter 20, pp. 935–1009.

Stewart, K. G. (1997), 'Exact testing in multivariate regression', *Econometric Reviews* **16**, 321–352.

Thode, Jr., H. C. (2002), *Testing for Normality*, number 164 *in* 'Statistics: Textbooks and Monographs', Marcel Dekker, New York.

Wilks, S. S. (1963), 'Multivariate statistical outliers', *Sankhya series A* **25**, 407–426.